Turkish Online Journal of Qualitative Inquiry (TOJQI) Volume 12, Issue 7, July 2021: 3621-3625

# **Certain Results on Pair Sum Labeling of Newly Constructed Graphs**

P. Noah Antony Daniel Renai, S. Roy

Department of MathematicsVellore Institute of TechnologyVellore, India Email: danielrenay@gmail.com

#### Abstract

For a (p,q) graph G, an injective map f from V(G) to  $\{\pm 1, \pm 2, ..., \pm p\}$  is said to be a pair sum labeling if the induced edge function  $f_e$  from E(G) to  $\{Z - 0\}$  defined by  $f_e(uv) = f(u) + f(v)$  is 1 - 1 and  $f_e(E(G))$  is either of the form  $\{\pm m_1, \pm m_2, ..., \pm m_q\}$  or  $\{\pm m_1, \pm m_2, ..., \pm m_{q-1}\} \cup$  $\{m_{q+1}\}$  depending on q which is either even or odd. A graph that admits the behavior of pair sum labeling is named a pair sum graph. In this manuscript we study the pair sum labeling of uniform (3, n)-cyclic graph, n-cyclic copies of  $C_3$  and path union of ladders.

**Keywords**: Pair Sum Labeling, Uniform, (3, n)-Cyclic Graph, *n*-Cyclic Copies of  $C_3$ , Path Union of Ladders.

#### 1. Introduction

Among the miscellaneous types of graph labeling, pair sum labeling of graphs is a newest form of labeling strategy. The labeling concept of pair sum graphs was introduced by Ponraj et al. [1]. The considered graph *G* in this article is of simple, undirected, finite. Terms not characterized here are utilized in the feeling of Harary [4]. An investigation of Pair sum labeling for certain standard graphs like cycle, path, bi-star complete graph, and some more kinds of graphs are discussed in [1-3]. Also they have proved most of all categories of trees admits pair sum labeling up to order  $n \ge 8$  [5].

## **Definition 1**

For a (p,q) graph G, an injective map f from V(G) to  $\{\pm 1, \pm 2, ..., \pm p\}$  is said to be a pair sum labeling if the induced edge function  $f_e$  from E(G) to  $\{Z - 0\}$  defined by  $f_e(uv) = f(u) + f(v)$  is 1 - 1 and  $f_e(E(G))$  is either of the form  $\{\pm m_1, \pm m_2, ..., \pm m_q\}$  or  $\{\pm m_1, \pm m_2, ..., \pm m_{q-1}\} \cup \{m_{q+1}\}$  depending on q which is either even or odd. A graph that admits the behavior of pair sum labeling is named a pair sum graph.

### 2. Main Result

## **Definition 2**

A uniform (3, n)-cyclic graph  $SC_3^n$  where *n* is even is obtained by attaching cycles of length 3 to all the pendent vertices of a star graph  $S_{n+1}$  (See Figure 2(a)).

## **Definition 3**

Let the graphs  $G_1, G_2, ..., G_n, n \ge 2$  be all replicated copies of a constant cycle graph G. Adding an edge in between any two vertices of  $G_i$  and  $G_{i+1}$  consecutively for i = 1, 2, ..., (n-1) and an edge between  $G_n$  and  $G_1$  is named as a uniform *n*-cyclic graph.

Let  $L_3^1, L_3^2, \ldots, L_3^m$  be *m* copies of ladder graph  $L_3$  are labeled as follows (See Figure. 1).



Figure. 1 m copies of ladder graph  $L_3$ 

### **Definition 4**

Let us consider the *m*-copies of ladder graphs  $L_3$  where  $m \ge 4$  is even. The graph  $P(L_3^m)$  is obtained by joining an edge in between the vertices  $v_i$  of  $L_3^i$  and  $u_{i+1}$  of  $L_3^{i+1}$  where i = 1, 2, ..., m - 1consecutively is named as path union of ladders.

### **Theorem 1**

The uniform (3, n)-cyclic graph  $SC_3^n$  is a pair sum graph if n is even.

### **Proof:**

Let the vertex set of  $SC_3^n$  be  $\{u_1, u_2, \dots, u_{\frac{3n}{2}}; 1 \le i \le n, v_1, v_2, \dots, v_{\frac{3n}{2}}; 1 \le i \le n\}$  and w be the hub vertex.

Now let us define  $f: V(SC_3^n) \to \{\pm 1, \pm 2, \dots, \pm \frac{3n}{2}\}$ 

such that

$$f(w) = 1$$

$$f(u_i) = 2i$$
 where  $i = 1, 2, ..., \frac{3n}{2}$   
 $f(v_i) = -2i$  where  $i = 1, 2, ..., \frac{3n}{2}$ 

Furtherly from the edge function which is induced, we have

$$\begin{split} f_e &: E(SC_3^n) \to \{Z - 0\}, \\ f_e(u_i u_j) &= 2i + 2j, i \neq j \end{split}$$

$$f_e(v_i v_j) = -2i - 2j, i \neq j$$

$$f_e(wu_i) = 1 + 2i,$$

$$f_e(wv_j) = 1 - 2j,$$
then

 $f(E(SC_3^n)) = \{\{\pm 3, \pm 9, \pm 15, \dots, \pm (3n-3)\}, \cup \{\pm 6, \pm 18, \pm 30, \dots, \pm (6n-6)\}, \cup \{\pm 8, \pm 20, \pm 32, \dots, \pm (6n-4)\}, \cup \{\pm 10, \pm 22, \pm 34, \dots, \pm (6n-2)\}\}.$ 

(See Figure 2(b))

Hence the graph  $SC_3^n$  is a pair sum graph.



Figure 2(a)  $SC_3^4$ 



Figure 2(b) Pair sum labeling of  $SC_3^4$ 

### **Theorem 2**

The *n*-cyclic graph  $C_3^n$ ,  $n \ge 2$  is a pair sum graph if *n* is even.

#### **Proof:**

Let the vertex set of  $C_3^n$  be  $\{u_1, u_2, \dots, u_{\frac{3n}{2}}; 1 \le i \le n, v_1, v_2, \dots, v_{\frac{3n}{2}}; 1 \le i \le n\}$ 

Now let us define  $f: V(\mathcal{C}_3^n) \to \{\pm 1, \pm 2, \dots, \pm \frac{3n}{2}\}$ 

such that

$$f(u_i) = i$$
 where  $i = 1, 2, ..., \frac{3n}{2}$   
 $f(v_i) = -i$  where  $i = 1, 2, ..., \frac{3n}{2}$ 

Furtherly from the edge function which is induced, we have

then

 $\begin{aligned} f(E(C_3^n)) &= \\ \{\pm 1, \pm 7, \pm 13, \dots, \pm (3n-5)\}, \cup \{\pm 3, \pm 9, \pm 15, \dots, \pm (3n-3)\}, \cup \{\pm 4, \pm 10, \pm 16, \dots, \pm (3n-2)\}, \{\pm 5, \pm 11, \pm 17, \dots, \pm (3n-1)\}, \}. \end{aligned}$ 

(See Figure 3(b))

Hence the graph  $C_3^n$  is a pair sum graph.



Figure 3(a)  $C_3^4$ 

Figure 3(b) Pair sum labeling of  $C_3^4$ 

### Theorem 3

The graph  $P(L_3^m)$  is a pair sum graph when  $m \ge 4$  is even.

### **Proof:**

Let the vertex set of  $P(L_3^m)$  be  $\{u_1, u_2, ..., u_{3m}, 1 \le i \le n, v_1, v_2, ..., v_{3m}, 1 \le i \le n\}$ 

Now let us define  $f: V(P(L_3^m)) \rightarrow \{\pm 1, \pm 2, \dots, \pm 3m\}$ 

such that

 $f(u_i) = i$  where i = 1, 2, ..., 3m

 $f(v_i) = -i$  where i = 1, 2, ..., 3m

Furtherly from the edge function which is induced, we have

$$f_e: E(P(L_3^m)) \to \{Z - 0\},\$$

$$\begin{aligned} f_e(u_i u_j) &= i + j, i \neq j \\ f_e(v_i v_j) &= -i - j, i \neq j \\ f_e(v_i u_{i+1}) &= 1 \end{aligned}$$

then

$$\begin{split} f(E(P(L_3^m)) &= 1 \cup \{\pm 3, \pm 15, \pm 27, \dots, \pm (6m-9)\} \cup \{\pm 4, \pm 16, \pm 28, \dots, \pm (6m-8)\} \cup \\ \{\pm 6, \pm 18, \pm 30, \dots, \pm (6m-6)\} \cup \{\pm 7, \pm 19, \pm 31, \dots, \pm (6m-5)\} \cup \{\pm 8, \pm 20, \pm 32, \dots, \pm (6m-4)\} \cup \\ \{\pm 10, \pm 22, \pm 34, \dots, \pm (6m-2)\} \cup \{\pm 11, \pm 23, \pm 35, \dots, \pm (6m-1)\} \cup \\ \{\pm 9, \pm 21, \pm 33, \dots, \pm (6m-15)\}. \end{split}$$

(See Figure (4(b))

Hence the graph  $P(L_3^m)$  is a pair sum graph.



### **3.**Conclusion

Therefore as a progress in this manuscript, we have determined pair sum labeling for certain graphs such as, uniform (3, n)-cyclic graph, uniform *n*-cyclic graph, path union of ladder graphs. Pair sum labeling for interconnection networks is under examination.

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