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Research Article

CYCLE RELATED GRAPHS - NEAR MEAN CORDIAL

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ABSTRACT

Let G = (V,E) be a simple graph. A **Near Mean Cordial Labeling** of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1} such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & \text{if}\big(f(u)+f(v)\big) \equiv 0 \ (\text{mod}2) \\ 0 & \text{else} \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.

In this paper, It is to be proved that , Tortoise graph T_n and Snail graph $S_n$are **Near Mean Cordial** graphs.

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Keywords and Phrases: Cordial labeling, Near Mean Cordial Labeling and Near Mean Cordial Graph.

I. INTRODUCTION

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology ,we referred Harary [4].For labeling of graphs, we referred Gallian[1]. A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge uv a label depending on the vertex labels of u and v.

A graph G is said to labeled if the n vertices are distinguished from one another by symbols such as $v_1, v_2, \ldots\ldots.. v_n$. In a labeling of a particular type, the vertices are assigned distinct values from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2]. In this paper, It is to be proved that Tortoise graph T_n and Snail graph S_n are **Near Mean Cordial** graphs.

II.PRELIMINARIES

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Definition 2.1 :

Let G= (V,E) be a simple graph. Let $f:V(G) \rightarrow \{0,1\}$ and the induced edge label,assigning $|f(u) - f(v)|$ is called a **Cordial Labeling** if the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the number of edges labeled 1differ by atmost 1 .A graph is called **Cordial** if it has a cordial labeling.

Definition 2.2:

Let G = (V,E) be a simple graph. G is said to be a **Mean Cordial Graph** if $f:V(G)\rightarrow\{0,1,2\}$ such that for each edge uv the induced map f*defined by $f^*(uv) = \left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where $\lfloor x \rfloor$ denote the least integer which is $\leq x$ and $|e_f(0)- e_f(1)|\leq 1$ where $e_f(0)$ is the number of edges with zero label. $e_f(1)$isthe number of edges with one label.

Definition 2.3:

Let G = (V,E) be a simple graph. A **Near Mean Cordial Labeling**of G is a function in $f : V(G) \rightarrow \{1, 2, 3, \ldots, p-1, p+1\}$ such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & \text{if}\big(f(u) + f(v)\big) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

and it satisfies the condition $|e_f(0)- e_f(1)|\leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.

Definition 2.4:

A **snail** $\mathbf{S_n}$ $(n \geq 4)$is obtained from path $P_n = v_1, v_2, \ldots, v_n$by attaching two parallel edges between v_i and v_{n-i+1} for $i = 1,2, \ldots, \left\lfloor \frac{n}{2} \right\rfloor$

Definition 2.5:

A **tortoise** T_n $(n > 4)$ is obtained from path $P_n = v_1, v_2, \ldots, v_n$by attaching one edges between v_i and v_{n-i+1} for $i = 1,2, \ldots, \left\lfloor \frac{n-1}{2} \right\rfloor$

III. MAIN RESULTS

Theorem 3.1: Snail graph (S_n) is a Near Mean Cordial Graph.

Proof:

Let $V(G) = \{u_i: 1 \leq i \leq n \}$

Let E(G) = {{(u_i u_{i+1}): $1 \le i \le n-1$} U{2(u_i u_{n-i+1}) : $1 \le i \le \frac{n}{2}$ }

Definef : V(G) → {1,2,3, …., n-1, n+1} by

Let $f(u_i) = 2i-1$, $1 \le i \le \frac{n}{2}$

$f(u_n) = n+1$

$f(u_{n/2+i}) = 2i$, $1 \le i \le \frac{n-2}{2}$

The induced edge labelings are,

$$f^*(u_iu_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$2f^*(u_iu_{n-i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{n-i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, 1 \le i \le \frac{n}{2}$$

Edge condition:-
Here, $e_f(0)$ =n and $e_f(1)$ = n-1
So, in all the cases, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.
Hence, S_n is a Near Mean Cordial graph.

For example, the Near Mean cordial labeling of S_8 is shown in Figure 3.1.1.

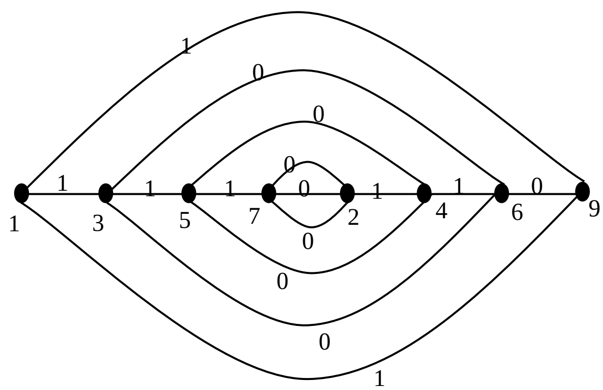

Figure 3.1.1

Theorem 3.2: Tortoise graph (T_n) is a Near Mean Cordial Graph.

Proof:

Let V(G) = { u_i: $1 \le i \le n$ }

Let E(G) = {{(u_i u_{i+1}): $1 \le i \le n-1$} U{ (u_i u_{n-i+1}) : $1 \le i \le \frac{n-1}{2}$ }

Define f : V(G) → {1,2,3, …., n-1, n+1} by

Let $f(u_{2i-1}) = i$, $1 \le i \le \frac{n-1}{2}$

$f(u_n) = n+1$

$f(u_{2i}) = n-2+(i-1)$, $\quad 1 \leq i \leq \frac{n-1}{2}$

The induced edge labelings are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, 1 \leq i \leq n-1$$

$$f^*(u_i u_{n-i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{n-i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, 1 \leq i \leq \frac{n-1}{2}$$

Edge condition:-

(i) If $n \equiv 1 \pmod 4$

Here, $e_f(0) = e_f(1) = \frac{3n-3}{4}$

(ii) If $n \equiv 3 \pmod 4$

Here, $e_f(0) = \frac{3n-1}{4}$ and $e_f(1) = \frac{3n-5}{4}$

So, in all the cases, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, T_n is a Near Mean Cordial graph.

For example, the Near Mean cordial labeling of T_7 and T_9 are shown in Figure 3.2.1.and 3.2.2.

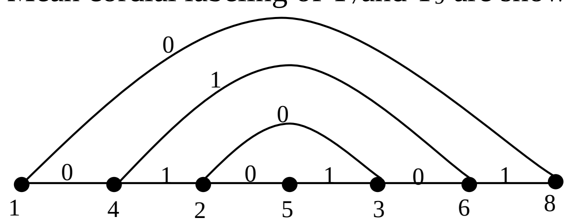

Figure 3.2.1.

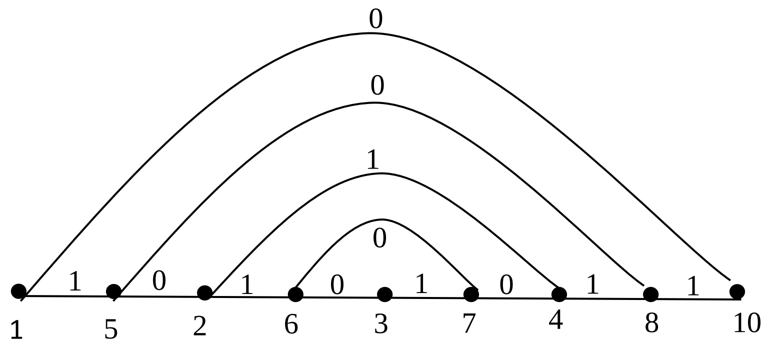

Figure 3.2.2.

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