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Research Article

I-Open(Closec) set and I-Continuous, I-Open(Closed) map in Hesitant Fuzzy Ideal Topological Space

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Abstract

In this paper, we introduce the definition of I- open (closed) set
in hesitant fuzzy ideal topological space and we prove some results about it. Also, we define the concept I-continuous and I-open (closed) map with prove results on them.

1.Introduction

The notion of fuzzy set was present by L.A. Zadeh [9] in 1965. Torra and Narukawa [8] in 2009 . Torra [7] in 2010 was introduced the notion of hesitant fuzzy set, that is a function form a set to a power set of the unit interval, the notion of hesitant fuzzy set was the other generalization of the notion fuzzy sets.

In 1966 K. Kuratowski [3] was the first introduced the basic concept "ideal topological space, local function and closure operators. M.E. Abd EL- Monsef, E.F. Lashien and A. A. Nase[1] in 1992 were introduced more new properties of I-openness. Also, were introduced new topological notions via ideals, I-closed sets, I- continuous function, I- open (closed) function.

J. Dontchev [2] In 1997 was studied the idea Hausdorff spaces via topological ideals and I- irresolute functions. D. Sarker [6] was introduced to fuzzy ideal in fuzzy theory and the concept of fuzzy local function. A.A. Salama and S.A .Alblowi [5] in 2012 were studied intuitionistic fuzzy ideals and intuitionisyic fuzzy local function.

J.G. Lee and K. Hur[ ] in 2020 defined hesitant fuzzy topological space and base, obtain some of their properties, respectively. Also, we introduce the concepts of a hesitant fuzzy neighborhood, closure, and interior and obtain some of their properties

Furthermore, we define a hesitant fuzzy continuous mapping and investigate some of its properties. Furthermore, we define a hesitant fuzzy subspace and obtain some of its properties.

Keywords: I-open (closed) set and I- continuous, I-open (closed) map, hesitant fuzzy ideal topological space.

2.Preliminaries

Definition(2.1)[3]

An ideal I on a set X is a non-empty collection of subsets of X which satisfies the following:

1) $A \in I$ and $B \subseteq A$ then $B \in I$.

2) $A \in I$ and $B \in L$ then $A \cup B \in I$.

If (X, τ) be a topological space and I an ideal on X, the triple (X, τ, I) is said to be ideal topological space.

Definition(2.2)[3]

An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X,τ,I). For a subset $A \subseteq X$, $A^*(I,\tau) = \{x \in X: A \cap \mu \notin I, \forall \mu \in N(x)\}$ is called the local function of A with respect to I and τ. We will write $A^*(I)$ or simply A^* for $A^*(I,\tau)$.

Definition(2.3)

Let X is a non- empty set and L anon- empty family of hesitant fuzzy sets. We will call L is a hesitant fuzzy ideal on X (HFI, for short) if:

1) If $h \in L$ and $k \subseteq h \Rightarrow k \in L$,
2) If $h \in L$ and $k \in L \Rightarrow h \cup k \in L$.

***Example* (2.4)**

Let X= {a, b} and h= {<a, {0.1, 0.3}>, <b, {0.2, 0.3}>}.

Then, $L = \{h^0, h, k : k \in HFL\ (X)$ and $k \subseteq h\}$is hesitant fuzzy ideal on X. ***Definition* (2.5)**

Let X be a reference set. Then:

1) If L_1={h^0}, then L_1 is said to be the smallest hesitant fuzzy ideal on X,
2) If $L_2$is all hesitant fuzzy set, then L_1 is said to be the largest hesitant fuzzy ideal on X.

***Definition* (2.6)**

Let (X, τ) be a hesitant fuzzy topological space and L be a hesitant fuzzy ideal on X, then (X, τ, L) is said to be a hesitant fuzzy ideal topological space (in short, $HFLTS$).

3.I- Open (Closed) in Hesitant Fuzzy Ideal Topological Space

***Definition* (3.1)**

Let (X, τ, L) be a hesitant fuzzy ideal topological space and $h \in HFS(X)$, h is said to be I −open if $h \subseteq int_H(h^*)$ we denoted by $IO(X,\tau) = \{ h \in HFS(X): h \subseteq int_H(h^*)\}$ or simply write IO for $IO(X,\tau)$. And h is said to be I −closed if its complement is $IO(X,\tau)$, [i.e.$h^c \in IO(X,\tau)$], it denoted by $IC(X,\tau)$.

***Example* (3.2)**

Let $X = \{x\}, \tau = \{h^0, h^1, k\}$ and $= \{h^0\}$, where $k(x) = [0.2, 0.5]$,

$h(x) = [0.4, 0.6]$ and $\alpha = \{0.2\}$

Since $h^*(L,\tau) = \cup \{x_\alpha \in H_P(X): h \cap U \notin L, U \in \tau\}$.

Then, $x_\alpha \in H_P(X)$implies $h^1 \cap h \notin L$ and $k \cap h \notin L$.

Then, $h^*(L,\tau) = cl_H(h)$

Since $cl_H(h) = \cap \{F$ is hesitant fuzzy closed set in : $h \subseteq F\}$.

Then, $HFCS(X) = \{h^1, h^0, k^c\}$, where $k^c(x) = [0, 0.2) \cup (0.5, 1]$

So, $h \subseteq h^1$ implies $cl_H(h) = h^1$, then $int_H(cl_H(h))= int_H(h^1)$

Since $int_H(h^1) = \cup \{U$ is hesitant fuzzy open set in τ: $U \subseteq h\}$.

Then, $h^0 \subseteq h^1, h^1 \subseteq h^1, k \subseteq h^1$ implies $int_H(h) = h^0 \cup h^1 \cup k = h^1$.

Hence $int_H(cl_H(h))= h^1$, then $h \subseteq int_H(h^*)$. Then, h is $I -$ open set.

***Proposition* (3.3)**

Let (X, τ, L) be a hesitant fuzzy ideal topological space and $h, k \in HFS(X)$. if $h \in IO(X, \tau)$ and $k \in \tau$, then $h \cap k \in IO(X, \tau)$.

Proof

Since $h \in IO(X, \tau)$, then $h \subseteq int_H(h^*)$.

Then $h \cap k \subseteq int_H(h^*) \cap k$.

Since $k \in \tau$. Implies $int_H(k) = k$

Hence $h \cap k \subseteq int_H(h^*) \cap int_H(k) = int_H(h^* \cap k)$.

So, $h \cap k \subseteq int_H(h^* \cap k)$,

by Theorem[$A^* \cap B \subseteq (A \cap B)^*$], then $h^* \cap k \subseteq (h \cap k)^*$

Hence $int_H(h^* \cap k) \subseteq int_H(h \cap k)^*$

So, $h \cap k \subseteq int_H(h^* \cap k) \subseteq int_H(h \cap k)^*$.Thus $h \cap k \subseteq int_H(h \cap k)^*$.

Hence $h \cap k \in IO(X, \tau)$.

Remark **(3.4)**

The intersection of two I −open sets need not be I −open set as the following example.

Example **(3.5)**

Let $X = \{x\}, \tau = \{h^0, h^1, h_1, h_2\}$ and $= \{h^0\}$, where

$h_1(x) = \{0.1, 0.2\}$, $h_2(x) = \{0.1, 0.2, 0.3\}$, $h(x) = \{0.1, 0.3\}$,

$k(x) = \{0.2, 0.3, 0.4\}, \alpha = \{0.1\}$.

Then, $h^*(L, \tau) = cl_H(h)$. Since $cl_H(h) = \cap \{F$ is $HFCS(X): h \subseteq F\}$.

$HFCS(X) = \{h^1, h^0, h_1^c, h_2^c\}$, such that $h_1^c = [0, 0.1) \cup (0.1, 0.2), \cup (0.2, 1]$,

$h_2^c = [0, 0.1) \cup (0.1, 0.2) \cup (0.2, 0.3) \cup (0.3, 1]$.

Then $h \subseteq h^1$ implies $cl_H(h) = h^1$

Since $int(h^1) = \cup \{U \in \tau: U \subseteq h^1\}$.

Then $h^0 \subseteq h^1, h^1 \subseteq h^1, h_1 \subseteq h^1, h_2 \subseteq h^1$

Thus, $int(h^1) = h^0 \cup h^1 \cup h_1 \cup h_2 = h^1$.

Hence $h \subseteq int_H(h^*)$. Then h is $I -$ open. To prove, k is $I -$ open.

Since $L = \{h^0\}$, then $k^*(L, \tau) = cl_H(k)$.

Since $cl_H(k) = \cap \{G$ is $S(X): k \subseteq G\}$. $HFCS(X) = \{h^1, h^0, h_1^c, h_2^c\}$.

Where, $h_1^c = [0, 0.1) \cup (0.1, 0.2), \cup (0.2, 1]$,

$h_2^c = [0, 0.1) \cup (0.1, 0.2) \cup (0.2, 0.3) \cup (0.3, 1]$.

Then $k \subseteq h^1$ implies $cl_H(k) = h^1$

Since $int(h^1) = \cup \{A \in \tau: A \subseteq h^1\}$.

Then $h^0 \subseteq h^1, h^1 \subseteq h^1, h_1 \subseteq h^1, h_2 \subseteq h^1$

Thus, $int(h^1) = h^0 \cup h^1 \cup h_1 \cup h_2 = h^1$.

Hence $k \subseteq int_H(h^*)$. Then k is $I -$ open.

So that $h(x) \cap k(x) = A(x) = \{0.3\}$, then $A^*(L, \tau) = cl_H(A)$

Hence $A \subseteq h_1^c$, $A \subseteq h^1$ implies $cl_H(A) = h_1^c \cap h^1 = h_1^c$.

Then $int_H(h_1^c) = h^0$ implies $A \nsubseteq int_H(A^*)$. Thus A is not $I -$ open

Remark **(3.6)**

The concept I −open sets and open sets are independent

4. I-Continuous and I-Open (Closed) mapping in Hesitant Fuzzy Ideal Topological Space

Definition **(4.1)**

A map $f:(X,\tau,I)\longrightarrow(Y,\psi)$ is said to be $I-$ continuous mapping if every open set $h\in\psi, f^{-1}(h)\in IO(X,\tau)$.

***Definition* (4.2)**

A map $f:(X,\tau,I)\longrightarrow(Y,\psi,J)$ is said to be $I-$irresolute continuous mapping if every $h\in IO(Y,\psi), f^{-1}(h)\in IO(X,\tau)$ and denoted by $Irr-$ continuous mapping.

***Definition* (4.3)**

A map $f:(X,\tau)\longrightarrow(Y,\psi,I)$ is said to be $I-$ open if every $h\in\tau$, $f(h)\in IO(Y,\psi)$.

***Definition* (4.4)**

A map $f:(X,\tau,I)\longrightarrow(Y,\psi,J)$ is said to be $I-$irresolute open if every $h\in IO(X,\tau), f(h)\in IO(Y,\psi)$ and denoted by $Irr-$ open.

***Definition* (4.5)**

A map $f(X,\tau)\longrightarrow(Y,\psi,I)$ is said to be $I-$ closed if every $h\in\tau$, $f(h)\in IC(Y,\psi)$.

***Definition*(4.6)**

A map $f:(X,\tau,I)\longrightarrow(Y,\psi,J)$ is said to be $I-$irresolute closed if every $h\in IC(X,\tau), f(h)\in IC(Y,\psi)$ and denoted by $Irr-$ closed.

***Proposition* (4.7)**

Let $f:(X,\tau_1,L_1)\longrightarrow(Y,\tau_2,L_2)$ is $Irr-$ continuous and $g:(Y,\tau_2,L_2)\longrightarrow(Z,\tau_3)$ is $I-$ continuous, then $g\circ f:(X,\tau_1,L_1)\longrightarrow(Z,\tau_3)$ is $I-$ continuous function.

Proof

Let $h\in\tau_3$ since g is I – continuous, then $g^{-1}(h)\in IO(Y,\tau_2)$.
Since f is Irr– continuous and $g^{-1}(h)\in IO(Y,\tau_2)$.
Then $f^{-1}(g^{-1}(h))\in IO(X,\tau_1)$.Since $f^{-1}\big(g^{-1}(h)\big)=(g\circ f)^{-1}(h)$
Hence $(g\circ f)^{-1}(h)\in IO(X,\tau_1)$.Thus $g\circ f$ is I – continuous.

***Example* (4.8)**

The identity mapping $I:(X,\tau,L)\longrightarrow(X,\tau,L)$ is $Irr-I$ – continuous. Let $h\in IO(X,\tau)$, then $f^{-1}(h)=h\in IO(X,\tau)$.

***Theorem* (4.9)**

Let $f:(X,\tau_1,L_1)\longrightarrow(Y,\tau_2)$ be a mapping, then the following are equivalent:

1) f is I – continuous.
2) For each $x_\alpha\in H_P(X)$ and $h\in\tau_2$ containing $f(x_\alpha)$, there exist $k\in IO(X,\tau_1)$ containing x_α such that $f(k)\subseteq h$.
3) For each $x_\alpha\in H_P(X)$ and each $h\in\tau_2$ containing $f(x_\alpha)$, $(f^{-1}(h))^*$ is a neighbor hood of x_α.

Proof (1)⟹ (2) Let $x_\alpha\in H_P(X)$ and $h\in\tau_2$ containing $f(x_\alpha)$,
By (1) f is I – continuous, then $f^{-1}(h)\in IO(X,\tau_1)$, put $f^{-1}(h)=k$
Then $k\in IO(X,\tau_1)$. Since $f(x_\alpha)\in h$ implies $x_\alpha\in f^{-1}(h)$, so $x_\alpha\in k$.
Then $f(k)=f\big(f^{-1}(h)\big)$ and $f\big(f^{-1}(h)\big)\subseteq h$. Hence $f(k)\subseteq h$.

Proof (2)⟹ (3) Let $x_\alpha\in H_P(X)$ and $h\in\tau_2$ containing $f(x_\alpha)$.
From (2) $x_\alpha\in f^{-1}(h)$, since f is I – continuous, then $f^{-1}(h)$ is I – open
Then $f^{-1}(h)\subseteq int_H(f^{-1}(h))^*\subseteq(f^{-1}(h))^*$
$x_\alpha\in f^{-1}(h)\subseteq(f^{-1}(h))^*$ implies $(f^{-1}(h))^*$ is neighborhood of x_α.

Proof (3)⟹ (1)

Let $x_\alpha \in H_P(X)$, $h \in \tau_2$ containing $f(x_\alpha)$ and $(f^{-1}(h))^*$ is neighborhood of x_α. Since $(f^{-1}(h))^*$ is neighborhood of x_α, there exists I –open set k such that $x_\alpha \in k \subseteq (f^{-1}(h))^*$, then $f(k) \subseteq f(f^{-1}(h))^* \subseteq h$.

***Theorem*(4.10)**

A function $f:(X,\tau_1,L_1) \longrightarrow (Y,\tau_2)$ is I –continuous if and only if the inverse image of each hesitant fuzzy closed set in τ_2 is I –closed.

Proof

Suppose that f is I – continuous and h hesitant fuzzy closed set in τ_2

Then h^c is hesitant fuzzy open set in τ_2, since f is I – continuous

So, $f^{-1}(h^c) \in IO(X,\tau_1)$, then $(f^{-1}(h))^c \in IO(X,\tau_1)$,

Thus $f^{-1}(h) \in I$ –closed in τ_1

Suppose that the inverse image of each hesitant fuzzy closed set in τ_2 is

I –closed, let $h \in \tau_2$

Implies h^c is hesitant fuzzy closed set in τ_2, then $f^{-1}(h^c) \in I$ –closed in τ_1

Then $f^{-1}(h^c) \in I$ –closed in τ_1 implies $f^{-1}(h))^c \in I$ –closed in τ_1

Hence $f^{-1}(h) \in IO(X,\tau_1)$.

***Theorem*(4.11)**

Let $f:(X,\tau_1,L_1) \longrightarrow (Y,\tau_2, L_2)$ is I – continuous and

$f^{-1}(h^*) \subseteq (f^{-1}(h))^*$. Then f is Irr – continuous.

Proof

Let $h \in IO(Y,\tau_2)$ implies $h \subseteq int_H(h^*)$. Then $f^{-1}(h) \subseteq f^{-1}(int_H(h^*))$, so $f^{-1}(h) \subseteq int_H(f^{-1}(h^*))$.

Since $f^{-1}(h^*) \subseteq (f^{-1}(h))^*$, then $int_H(f^{-1}(h^*)) \subseteq int_H(f^{-1}(h))^*$

Then $f^{-1}(h) \subseteq int_H(f^{-1}(h^*)) \subseteq int_H(f^{-1}(h))^*$.

Hence $f^{-1}(h) \subseteq int_H(f^{-1}(h))^*$. Then $f^{-1}(h) \in IO(X,\tau_1)$.

***Theorem* (4.12)**

Let $f:(X,\tau_1,L_1) \longrightarrow (Y,\tau_2,L_2)$ and $g:(Y,\tau_2,L_2) \longrightarrow (Z,\tau_3)$, if f is surjection, $(g \circ f)^{-1}(h^*) \subseteq ((g \circ f)^{-1}(h))^*$ and both f and g are I – continuous then $g \circ f$ is I – continuous.

Proof

Let h is hesitant fuzzy open set in τ_3, since g is I – continuous,

then $g^{-1}(h) \in IO(Y,\tau_2)$ implies $g^{-1}(h) \subseteq int_H(g^{-1}(h))^*$.

Then $f^{-1}\big(g^{-1}(h)\big) \subseteq f^{-1}(int_H(g^{-1}(h))^*)$.

Implies $f^{-1} \circ g^{-1}(h) \subseteq int_H(f^{-1}\big(g^{-1}(h^*)\big)$.

Then $f^{-1} \circ g^{-1}(h) \subseteq int_H\big(f^{-1} \circ g^{-1}(h^*)\big)$.

So, $(g \circ f)^{-1}(h) \subseteq int_H(g \circ f)^{-1}(h)) \subseteq int_H((g \circ f)^{-1}(h))^*$

Then $(g \circ f)^{-1}(h) \subseteq int_H((g \circ f)^{-1}(h))^*$

Hence$(g \circ f)^{-1}(h) \in IO(X,\tau_1)$.

***Theorem* (4.13)**

Let $f:(X,\tau_1,L_1)\longrightarrow(Y,\tau_2,L_2)$ and $g:(Y,\tau_2,L_2)\longrightarrow(Z,\tau_3,L_3)$ are two $I-$open function, then $g\circ f:(X,\tau_1,L_1)\longrightarrow(Z,\tau_3,L_3)$ is $I-$open.

Proof

Let h is hesitant fuzzy open set in τ_1.

Since f is $I-$open function, then $f(h)\in IO(Y,\tau_2)$.

Since g is $I-$open function and $f(h)\in IO(Y,\tau_2)$, then $g(f(h))\in IO(Z,\tau_3)$,

Then $g\circ f(h))\in IO(Z,\tau_3)$. Hence $g\circ f$ $I-$open function.

***Theorem* (4.14)**

Let $f:(X,\tau_1)\longrightarrow(Y,\tau_2,L)$ is $I-$open if for each $x_\alpha\in H_P(X)$ and each neighborhood of x_α there exist $I-$open set $k\in HFS(X)$ containing $f(x_\alpha)$ such that $k\subseteq f(h)$

Proof

Let $x_\alpha\in H_P(X)$ and $\in\ \tau_1$. Since f is $I-$open, then $f(k)\in IO(Y,\tau_2)$.

So that, $f(k)\ \subseteq int_H(f^{-1}(k))^*$.

Since there exists neighborhood of x_α then $x_\alpha\in k\subseteq h$.

Since $x_\alpha\in k$, then $f(x_\alpha)\in f(k)$. So that, $f(k)\subseteq f(h)$. Implies $A=f(k)\subseteq f(h)$.

***Theorem*(4.15)**

For any bijective function $f:(X,\tau_1)\longrightarrow(Y,\tau_2,L)$ the following are equivalent:

(1) $f^{-1}:(Y,\tau_2,L)\longrightarrow(X,\tau_1)$ is $I-$ continuous

(2) f is $I-$ open.

(3) f is $I-$ closed.

Proof $(\mathbf{1})\Longrightarrow(\mathbf{2})$

Suppose that f^{-1} is $I-$ continuous and h is hesitant fuzzy open set in τ_1

Then $(f^{-1})^{-1}(h)\in IO(Y,\tau_2)$, so $f(h)\in IO(Y,\tau_2)$. Hence f is $I-$ open.

Proof $(\mathbf{2})\Longrightarrow(\mathbf{3})$

Suppose that f is $I-$ open and h is hesitant fuzzy closed set in τ_1

Then h^c is hesitant fuzzy open set in $\tau_1$since f is $I-$ open.

So, $f(h^c)\ \in IO(Y,\tau_2)$ implies $(f(h))^c\ \in IO(Y,\tau_2)$

Hence $f(h)\in I-$ closed in τ_2. Thus f is $I-$ closed.

Proof $(\mathbf{3})\Longrightarrow(\mathbf{1})$

Suppose that f is $I-$ closed and h is hesitant fuzzy open set in τ_1.

Then h^c is hesitant fuzzy closed set in τ_1, since f is $I-$ closed.

So $f(h^c)\in\ I-$ closed in τ_2 implies $(f(h))^c\ \in\ I-$ closed in τ_2

Hence $f(h)\in IO(Y,\tau_2)$, since $(f^{-1})^{-1}(h)=\ f(h)$.

Then $(f^{-1})^{-1}(h)\in IO(Y,\tau_2)$. Thus f^{-1} is $I-$ continuous.

***Theorem* (4.16)**

If $f:(X,\tau_1,L_1)\longrightarrow(Y,\tau_2,L_2)$ is $Irr-$ open and for each $h\in HFS(S)$,

$f(h^*)\subset\ [f(h)]^*$, then the image of each $I-$ open set is $I-$ open.

Proof

Suppose that $h\in IO(X,\tau_1)$, then $h\subseteq int_H(h^*)$

Implies $f(h)\subseteq f(int_H(h^*))$. Then $f(h)\subseteq int_H\big(f(h^*)\big)$.

Since $int_H(f(h^*))\subseteq int_H\ (f(h))^*$.

Then $f(h)\subseteq int_H\ \big(f(h)\big)^*$. Hence $f(h)\in IO(Y,\tau_2)$.

***Theorem* (4.17)**

Let $f:(X,\tau_1,L_1)\longrightarrow(Y,\tau_2,L_2)$ and $g:(Y,\tau_2,L_2)\longrightarrow(Z,\tau_3,L_3)$ are two functions where L_1, L_2, L_3 are ideals on X, Y and Z respectively. Then:

1) $g\circ f$ is I −open, if f is open function and g is I −open.
2) f is I −open if $g\circ f$ is open and g is $I-$ continuous.
3) If f and g are two I −open, f is surjective and $g(h^*)\subset[g(h)]^*$ for each $h\in HFS(X)$, then $g\circ f$ is I −open.

Proof (1)

Suppose that $g\circ f:(X,\tau_1,L_1)\longrightarrow(Z,\tau_3,L_3)$ and h is hesitant fuzzy open set in τ_1, since f is open, then $f(h)$ is hesitant fuzzy open set in τ_2. Since g is I −open and $f(h)\in\tau_2$, then $g(f(h)\in IO(Z,\tau_3)$. Hence $g\circ f(h)\in IO(Z,\tau_3)$. Thus, $g\circ f$ is I −open.

Proof (2)

Let h is hesitant fuzzy open set in τ_1, since $g\circ f$ is open function.
Then $g\circ f(h)$ is hesitant fuzzy open set in τ_3, since g is $I-$ continuous.
Implies $g^{-1}\ (g\circ f)(h)\in IO(Y,\tau_2)$, then $f(h)\in IO(Y,\tau_2)$.Thus f is I −open.

Proof (3)

Let $h\ \in\tau_1$. Since f is I −open, then $(h)\in IO(Y,\tau_2)$.
Implies $f(h)\ \subseteq int_H(f(h))^*$.
Then $g(f(h))\ \subseteq g(int_H(f(h))^*)$. So, $(g\circ f)(h)\subseteq\ int_H(g(f(h))^*)$
$(g\circ f)(h)\subseteq\ int_H(g(f(h^*)))$, so $(g\circ f)(h)\subseteq\ int_H(g\circ f)(h^*)$
$(g\circ f)(h)\in IO(Z,\tau_3)$. Then $g\circ f$ is I −open.

***Theorem* (4.18)**

Let $f:(X,\tau_1,L_1)\longrightarrow(Y,\tau_2,L_2)$ and $g:(Y,\tau_2,L_2)\longrightarrow(Z,\tau_3,L_3)$ are two functions where L_1, L_2, L_3 are ideals on X, Y and Z respectively. Then:

1) $g\circ f$ is I −continuous, if f is continuous function and g is $I-$ continuous.
2) f is $I-$ continuous if $g\circ f$ is continuous and g is $I-$ open.

Proof (1)

Let $g\circ f:(X,\tau_1,L_1)\longrightarrow(Z,\tau_3,L_3)$ and $h\in\tau_3$.
Since g is $I-$ continuous, $g^{-1}(h)\in IO(Y,\tau_2)$.
Then f is continuous, then $f^{-1}(g^{-1}(h))\in IO(X,\tau_1)$.
Thus, $(g\circ f)^{-1}(h)\in IO(X,\tau_1)$.
Hence $g\circ f$ is I −open.

Proof (2)

Let $h\in\tau_2$, since g is I −open, then $g(h)\in IO(Z,\tau_3)$.
Since $g\circ f(h)$ is continuous, then $(g\circ f)^{-1}(g(h))\ \in IO(X,\tau_1)$.
Then, $f^{-1}(g^{-1}(g(h))\in IO(X,\tau_1)$.
Implies $f^{-1}(h)\in IO(X,\tau_1)$.
Thus, f is I −open.

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