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Research Article

Decomposition Of $(A_{\mathrm{h}}, \Lambda)$ -Continuity

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Abstract

In this paper, we introduce and study the notions of αg_μ - H -closed sets and $(\alpha g_{\mathrm{H}}, \lambda)$ - continuous functions in hereditary generalized topological spaces. Also we obtain a decomposition of $(\alpha_{\mathrm{H}}, \lambda)$ -continuity and $(\sigma_{\mathrm{H}}, \lambda)$ -continuity on a hereditary generalized topological space.

1 Introduction

In the year 2002, Csaszar [1] introduced very usefull notions of generalized topology $(G.T.)$ and generalized continuity. A subset A of a space (Z, μ) is μ - α *-open* [2], if $A \subset i_\mu c_\mu i_\mu(A)$. Let us denote by $\alpha(\mu)$ that of all μ - α -open sets. The μ - α *-interior* [2] of a subset A of a $G.T.S.$ (Z, μ) denote by $i_\alpha(A)$,is defined by the union of all μ - α -open sets of (Z, μ) contained A. A subset$A$ of (Z, μ) is said to be αg_μ -closed [5], if $c_\alpha(A) \subset M$ whenever $A \subset M$ and M is μ -open in (X, μ) . A nonempty family H of subsets of Z is said to be a *hereditary class* [3], if $A \in \mathrm{H}$ and $M \subset A,$ then $M \in \mathrm{H}.$ A $G.T.S.$ (Z, μ) with a hereditary class H is hereditary generalized topological space $(H.G.T.S.)$ and denoted by (Z, μ, H). For each $A \subseteq X,$ $A^*(\mathrm{H}, \mu) = \{z \in X : A \cap M \notin \mathrm{H}$ for every $M \in \mu$ such that $z \in M\}$ [3]. For $A \subset Z,$ define $c^*_\mu(A) = A \cup A^*(\mathrm{H}, \mu)$ and $\mu^* = \{A \subset Z : Z - A = c^*_\mu(Z - A)\}$. Let A be a subset of $H.G.T.S.$ (Z, μ, H) is α - H -open [3]), if $A \subset i_\mu c^*_\mu i_\mu(A))$. A map $f : (Z, \mu) \to (W, \lambda)$ is (μ, λ) *-continuous*

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[1] (resp. (σ,λ)-*continuous* [4], (α,λ)-*continuous* [4]), if for each λ-open set M in W, $f^{-1}(M)$ is μ-open (resp. μ-σ-open, μ-α-open) in (Z,μ).

Lemma 1.1. [[2], Lemma 2.2] Let (Z,μ) be a $G.T.S.$ For any $L \subset Z$, we have

1. $i_\alpha(L) = L \cap i_\mu c_\mu i_\mu(L)$.

2 αg_μ-H-closed

Definition 2.1. *A subset A of a hereditary generalized topological space (X,μ,H) is said to be α-H-closed, if A^c is α-H-open.*

Definition 2.2. *Let A be a subset of a hereditary generalized topological space (X,μ,H). Then $i_{\alpha\mathrm{H}}(A)$ is the union of all α-H-open set contained in A.*

Propositon 2.3. *Let A be a subset of a hereditary generalized topological space (X,μ,H). Then $i_{\alpha\mathrm{H}}(A) = A \cap i_\mu c^*_\mu i_\mu(A)$.*

Proof. Let A be a subset of a hereditary generalized topological space (X,μ,H).
Then $A \cap i_\mu c^*_\mu i_\mu(A) \subset i_\mu c^*_\mu i_\mu(A)$
$= i_\mu(c^*_\mu(i_\mu(A \cap i_\mu(c^*_\mu(i_\mu(A))))))$
$= i_\mu(c^*_\mu(i_\mu(i_\mu(A) \cap c^*_\mu(i_\mu(A)))))$
$= i_\mu(c^*_\mu(i_\mu(i_\mu(A)))) \cap i_\mu(c^*_\mu(i_\mu(A)))$
$\subseteq i_\mu(c-\mu^*(i_\mu(A \cap i_\mu(c^*_\mu(i_\mu(A))))))$.
Hence $A \cap i_\mu(c^*_\mu(i_\mu(A)))$ is an α-H-open in (X,μ,H) and contained in A. Thus, $A \cap i_\mu(c^*_\mu(i_\mu(A))) \subset i_{\alpha\mathrm{H}}(A)$.
Now $i_{\alpha\mathrm{H}}(A)$ is α-H-open in (X,μ,H). Therefore
$i_{\alpha\mathrm{H}}(A) \subset i_{\alpha\mathrm{H}}(A) \cap i_\mu(c^*_\mu(i_\mu(A)))$
$\subseteq A \cap i_\mu(c^*_\mu(i_\mu(A)))$.
Hence $i_{\alpha\mathrm{H}}(A) = A \cap i_\mu c^*_\mu i_\mu(A)$.

Definition 2.4. *Let A be a subset of a hereditary generalized topological space (X,μ,H). Then $c_{\alpha\mathrm{H}}(A)$ is intersection of all α-H-closed set containig A.*

Propositon 2.5. *Let A be a suset of a hereditary generalized topological space (X, μ, H). Then* $c_{\alpha\mathrm{H}}(A) = A \cup c_\mu i^*_\mu c_\mu(A)$.

Proof. Let A be a suset of a hereditary generalized topological space (X, μ, H). Then

$c_\mu(i^*_\mu(c_\mu(A \cup c_\mu(i^*_\mu(c_\mu(A)))))) = c_\mu(i^*_\mu(c_\mu(A))) \cup c_\mu(i^*_\mu(c_\mu(A)))$

$= c_\mu(i^*_\mu(c_\mu(A)))$

$\subseteq A \cup c_\mu(i^*_\mu(c_\mu(A)))$.

Now, $A \cup c_\mu(i^*_\mu(c_\mu(A)))$ is α-H-closed.

Hence $c_{\alpha\mathrm{H}}(A) \subseteq c_\mu(i^*_\mu(c_\mu(A)))$.

Now, $c_\mu(i^*_\mu(c_\mu(A))) \subseteq c_\mu(i^*_\mu(c_\mu(c_{\alpha\mathrm{H}}(A))) \subseteq c_{\alpha\mathrm{H}}(A)$.

$\Rightarrow$ $A \cup c_\mu(i^*_\mu(c_\mu(A) \subseteq A \cup c_{\alpha\mathrm{H}}(A) = c_{\alpha\mathrm{H}}(A)$ Thus, $A \cup c_{\alpha\mathrm{H}}(A) \subseteq c_{\alpha\mathrm{H}}(A)$.

Hence $c_{\alpha\mathrm{H}}(A) = A \cup c_\mu i^*_\mu c_\mu(A)$.

Definition 2.6. *Let (X, μ, H) be a hereditary generalized topological space. A subset A of X is said to be αg_μ-H-closed if $c_{\alpha\mathrm{H}}(A) \subseteq M$ whenever $A \subseteq M$ and M isμ-open.*

Theorem 2.7. *Every μ-closed set is αg_μ-H-closed set but not conversely.*

Proof. Let $A \subset X$ is μ-closed such that $A \subseteq M$ and M is μ-open. Now $c_{\alpha\mathrm{H}}(A) \subset c_\mu(A) \subseteq M$ and M is μ-open. Hence A is αg_μ-H-closed set

Example 2.8. *Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, d, e\}$, $\{a, b, d, e\}, \{a, c, d, e\}, X\}$ and $\mathrm{H} = \{\emptyset, \{c\}\}$. Then $A = \{a, c, d\}$ is αg_μ-H-closedset but not μ-closed.*

Theorem 2.9. *Every α-H-closed set is αg_μ-H-closed set but not conversely.*

Proof. Let $A \subset X$ is α-H-closed. Consider M be any μ-open set and $A \subseteq M$. Since A is α-H-closed, so $c_{\alpha\mathrm{H}}(A) \subseteq M$ whenever $A \subseteq M$ and M is μ-open. Hence A is αg_μ-H-closed.

Example 2.10. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, d, e\},$ $\{a, b, d, e\}, \{a, c, d, e\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{c\}\}$. *Then* $A = \{a, c, d\}$ *is* αg_μ - H*-closedset but not* α - H*-closed set.*

Remark 2.11. *The intersection of any two* αg_μ - H*-closed sets need not be* αg_μ - H*-closed set.*

Example 2.12. *Let* $X = \{a, b, c\}$, $\mu = \{\emptyset, \{a\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{c\}\}$. *Then* $A = \{a, b\}$ *and* $B = \{a, c\}$ *are two* αg_μ - H*-closed sets but* $A \cap B = \{a\}$ *is not a* αg_μ - H*-closed set.*

Theorem 2.13. *If a subset* A *of* X *is* αg_μ - H *-closed in* (X, μ, H), *then* $c_{\alpha\mathrm{H}}(A) - A$ *contains no nonempty* μ *-closed sets of* (X, μ).

Proof. Assume that A is αg_μ - H -closed. Let F be a non empty μ -closed set contained in $c_{\alpha\mathrm{H}}(A) - A$. Since $A \subseteq X - F$ and A is αg_μ - H -closed, $c_{\alpha\mathrm{H}}(A) \subseteq X - F$ and $F \subseteq X - c_{\alpha\mathrm{H}}(A)$. Therefore, $F \subseteq c_{\alpha\mathrm{H}}(A) \cap (X - c_{\alpha\mathrm{H}}(A)) = \emptyset$, which implies that $c_{\alpha\mathrm{H}}(A) - A$ contains no nonempty μ -closed sets.

Corollary 2.14. *Let* (X, μ) *be a strong generalized topological space with hereditary class* H *and* $A \subset X$ *is* αg_μ - H*-closed. Then* A *is* α - H*-closed iff* $c_{\alpha\mathrm{H}}(A) - A$ *is* μ *-closed.*

Proof. Let A be α - H -closed. If A is α - H -closed $c_{\alpha\mathrm{H}}(A) - A = \emptyset$ and $c_{\alpha\mathrm{H}}(A) - A$ is μ -closed. Conversely, let $c_{\alpha\mathrm{H}}(A) - A$ be μ -closed set, where A is α - H -closed. Then by Theorem 2.13 , $c_{\alpha\mathrm{H}}(A) - A$ does not contain any non empty μ -closed set. Since $c_{\alpha\mathrm{H}}(A) - A$ is a μ -closed subset of itself, $c_{\alpha\mathrm{H}}(A) - A = \emptyset$ and hence A is α - H-closed.

Theorem 2.15. *If* A *is* μ *-open and* αg_μ - H*-closed in* (X, μ, H) *then* A *is* α - H*-closed in* (X, μ).

Proof. Let A be a μ -open and αg_μ - H-closed in (X, μ, H). Then $c_{\alpha\mathrm{H}}(A) \subseteq A$ and hence A is α - H-closed in (X, μ).

Propositon 2.16. *A subset* A *of* X *is* αg_μ - H*-closed in* (X, μ, H) *if and only if* $c_{\alpha\mathrm{H}}(A) \cap F = \emptyset$ *whenever* $A \cap F = \emptyset$ *and* F *is* μ *-closed in* (X, μ, H).

Proof. Assume that A is αg_μ - H -closed. Let $A \cap F = \emptyset$ and F is μ -closed. Then $A \subseteq X - F$ and $c_{\alpha\mathrm{H}}(A) \subseteq X - F$. Therefore, we have $c_{\alpha\mathrm{H}}(A) \cap F = \emptyset$. Conversely, let $A \subseteq M$ and M be μ -open. Then $A \cap (X - M) = \emptyset$ and $X - M$ is μ -closed. By hypothesis, $c_{\alpha\mathrm{H}}(A) \cap (X - M) = \emptyset$ and hence $c_{\alpha\mathrm{H}}(A) \subseteq M$. Therefore, A is an αg_μ - H-closed set.

Theorem 2.17. *For a subset* A *of* (X, μ), *the following properties are equivalent:*

1. *A is μ -locally closed,*
2. *$A = U \cap c_{\alpha\mathrm{H}}(A)$ for some $U \in \mu$,*

3. $c_{\alpha\mathrm{H}}(A) - A$ *is* μ*-closed,*

4. $A \cup (X - c_{\alpha\mathrm{H}}(A)) \in \mu$,

5. $A \subset i_{\alpha\mathrm{H}}(A \cup (X - c_{\alpha\mathrm{H}}(A)))$.

Proof. (1) ⇒ (2). Let $A = U \cap V$, where $U \in \mu$ and V is μ-closed. Since $A \subset V$, we have $c_{\alpha\mathrm{H}}(A) \subset c_{\alpha\mathrm{H}}(V) \subset c_{\mu}(V) = V$. Since $A \subset U \cap c_{\alpha\mathrm{H}}(A) \subset U \cap V = A$. Therefore, we obtain $A = U \cap c_{\alpha\mathrm{H}}(A)$ for some $U \in \mu$.

(2) ⇒ (3). Suppose that $A = U \cap c_{\alpha\mathrm{H}}(A)$ for some $U \in \mu$. Then, $c_{\alpha\mathrm{H}}(A) - A = c_{\alpha\mathrm{H}}(A) \cap [X - (U \cap c_{\alpha\mathrm{H}}(A))] = c_{\alpha\mathrm{H}}(A) \cap (X - U)$. Since $c_{\alpha\mathrm{H}}(A) \cap (X - U)$ is μ-closed and hence, $c_{\alpha\mathrm{H}}(A) - A$ is μ-closed.

(3) ⇒ (4). We have $X - (c_{\alpha\mathrm{H}}(A) - A) = (X - c_{\alpha\mathrm{H}}(A)) \cup A$ and hence, by (3) we obtain $A \cup (X - c_{\alpha\mathrm{H}}(A)) \in \mu$.

(4) ⇒ (5). By (4), $A \subset A \cup (X - c_{\alpha\mathrm{H}}(A)) = i_{\alpha\mathrm{H}}(A \cup (X - c_{\alpha\mathrm{H}}(A)))$.

(5) ⇒ (1). Let $U = i_{\alpha\mathrm{H}}[A \cup (X - c_{\alpha\mathrm{H}}(A))]$. Then, $U \in \mu$ and $A = A \cap U \subset U \cap c_{\mu}(A) \subset [A \cup (X - c_{\alpha\mathrm{H}}(A))] \cap c_{\mu}(A) = A \cap c_{\mu}(A) = A$. Therefore, we obtain $A = U \cap c_{\mu}(A)$, where $U \in \mu$ and $c_{\mu}(A)$ is μ-closed. Hence A is μ-locally closed.

Theorem 2.18. *Let A and B be subsets of a hereditary generalized topologicalspace* (X, μ, H). *If $A \subset B \subset c_{\alpha\mathrm{H}}(A)$ and A is αg_{μ}-H-closed, then B is αg_{μ}-H-closed.*

Proof. Assume that $A \subset B \subset c_{\alpha\mathrm{H}}(A)$ and A is αg_{μ}-H-closed. Then we have $c_{\alpha\mathrm{H}}(B) - B \subset c_{\alpha\mathrm{H}}(A) - A$. Let F be a μ-closed set such that $F \subset c_{\alpha\mathrm{H}}(B) - B \subset c_{\alpha\mathrm{H}}(A) - A$. Since A is αg_{μ}-H-closed, therefore $c_{\alpha\mathrm{H}}(A) - A$ has no non-empty μ-closed subset and hence $c_{\alpha\mathrm{H}}(B) - B$ contains no nonempty μ-closed subset. Hence$B$ is αg_{μ}-H-closed.

Theorem 2.19. *If a subset A of (X, μ, H) is αg_{μ}-H-closed and B is μ-closed,then $A \cap B$ is αg_{μ}-H-closed.*

Proof. Suppose that $A \cap B \subseteq M$, where M is μ-open in (X, μ, H). Then $A \subseteq (M \cup (X - B))$. Since A is αg_{μ}-H-closed, $c_{\alpha\mathrm{H}}(A) \subseteq M \cup (X - B)$ and hence $c_{\alpha\mathrm{H}}(A) \cap B \subseteq M$. Therefore, $c_{\alpha\mathrm{H}}(A \cap B) \subseteq M$ which implies that $A \cap B$ is αg_{μ}-H-closed.

Definition 2.20. *A subset A of a hereditary generalized topologicalspace* (X, μ, H) *is αg_{μ}-H-open if and only if A^c is αg_{μ}-H-closed.*

Theorem 2.21. *A subset A of a hereditary generalized topological space* (X, μ, H) *is αg_{μ}-H-open if and only if $F \subset \alpha\mathrm{H}i_{\mu}(A)$ whenever F is μ-closed and $F \subset A$.*

Proof. Assume that $F \subset \alpha\mathrm{H}i_{\mu}(A)$ whenever F is μ-closed and $F \subset A$. Let $A^c \subset M$, where M is μ-open. Then $M^c \subset A$ and M^c is μ-closed, therefore $M^c \subset \alpha\mathrm{H}i_{\mu}(A)$, which implies $\alpha\mathrm{H}c_{\mu}(A^c) \subset M$. So A^c is αg_{μ}-H-closed. Hence A is αg_{μ}-H-open.

Conversely, suppose that A is αg_μ-H-open, $F \subset A$ and F is μ-closed. Then F^c is open and $A^c \subset F^c$. Therefore, $\alpha\mathrm{H}c_\mu(A^c) \subset F^c$ and so $F \subset \alpha\mathrm{H}i_\mu(A)$.

Theorem 2.22. *Every αg_μ-H-open set is αg_μ-open set but not conversely.*

Proof. Let $A \subset X$ is αg_μ-H-open in (X, μ, H). Then we have, $F \subset \alpha\mathrm{H}i_\mu(A)$ whenever $F \subset A$ and F is μ-closed in (X, μ, H).

Since $F \subset \alpha\mathrm{H}i_\mu(A) = A \cap i_\mu c^*_\mu i_\mu(A)$

$\subseteq i_\mu c_\mu i_\mu(A)$

$= i_\alpha(A)$.

Hence A is αg_μ-open set by Theorem 2.11 [5].

Theorem 2.23. *Let A and B be subsets of a hereditary generalized topological space (X, μ, H). If $i_{\alpha\mathrm{H}}(A) \subset B \subset A$ and A is αg_μ-H-open, then B is αg_μ-H-open.*

Proof. Suppose that $i_{\alpha\mathrm{H}}(A) \subset B \subset A$. Then $X - A \subset X - B \subset c_{\alpha\mathrm{H}}(X - A)$. ByTheorem 2.18, $X - B$ is αg_μ-H-closed. Hence B is αg_μ-open.

Propositon 2.24. *Let (X, μ) be a strong generalized topological space with hereditary class H. For each $x \in X$, either $\{x\}$ is μ-closed or $\{x\}$ is αg_μ-H-open.*

Proof. Let $\{x\}$ be not μ-closed. Then $X - \{x\}$ is not μ-open and the only μ-open set containing $X - \{x\}$ is X itself. Therefore $c_{\alpha\mathrm{H}}(X - \{x\}) \subseteq X$ and hence $X - \{x\}$ is αg_μ-H-closed. Thus $\{x\}$ is αg_μ-H-open.

Remark 2.25. *The notions of μ^*-closed and αg_μ-H-closed are independent.*

Example 2.26. *Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, d, e\}, \{a, b, d, e\}, \{a, c, d, e\}, X\}$ and $\mathrm{H} = \{\emptyset, \{c\}\}$. Then $A = \{a, c, d\}$ is αg_μ-H-closedset but not μ^*-closed.*

Example 2.27. *Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset. \{b\}, \{c, d\}, \{b, c, d\}, X\}$ and $\mathrm{H} = \{\emptyset, \{c\}\}$. Then $A = \{c\}$ is μ^*-closed but not αg_μ-H-closed set.*

Definition 2.28. *A subset A of a hereditary generalized topological space (X, μ, H) is said to be H-R-closed, if $A = c^*_\mu i_\mu(A)$.*

Theorem 2.29. *Every H-R-closed is σ-H-open but not conversely.*

Proof. Let $A \subset X$ is H-R-closed in (X, μ, H). Then $A = c^*_\mu i_\mu(A)$, which implies $A \subset c^*_\mu i_\mu(A)$. Hence A is σ-H-open.

Example 2.30. *Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a, b\}, \{c\}, \{a, b, c\}, \{b, c, d\}, X\}$ and*

$\mathrm{H} = \{\emptyset, \{d\}\}$. *Then* $A = \{c\}$ *is* σ-H*-open but not* H-R*-closed.*

Definition 2.31. *A subset* A *of a hereditary generalized topologicalspace* (X, μ, H) *is said to be*

1. *a* η_μ-H*-set, if* $A = U \cap V$*, where* U *is* μ*-open and* V *is* α-H*-closed.*
2. *a* $s\mathrm{A}_{\mathrm{H}}$*-set, if* $A = U \cap V$*, where* U *is* σ-μ*-open and* V *is* H-R*-closed.*

Remark 2.32. *The union of two* η_μ-H*-sets need not be a* η_μ-H*-set.*

Example 2.33. *Let* $X = \{\varsigma_1, \varsigma_2, \varsigma_3\}$, $\mu = \{\emptyset, \{\varsigma_1\}, \{\varsigma_1, \varsigma_2\}, X\}$ *and* $\mathrm{H} = \{\emptyset, \{\varsigma_2\}, \{\varsigma_3\}, \{\varsigma_2, \varsigma_3\}\}$. *Then* $A = \{\varsigma_1\}$ *and* $B = \{\varsigma_3\}$ *are* η_μ-H*-sets but* $A \cup B = \{\varsigma_1, \varsigma_3\}$ *is not a* η_μ-H*-set.*

Theorem 2.34. *Let A and B be a subset of quasi topological space* (X, μ) *withhereditary class* H. *If A and B are* η_μ-H*-sets. Then* $A \cap B$ *is also an* η_μ-H*-set.*

Proof. Let $A = U \cap V$ and $B = L \cap M$, where U and L are μ-open sets and V and M are α-H-closed sets. Now $A \cap B = (U \cap V) \cap (L \cap M) = (U \cap L) \cap (V \cap M)$,where $(U \cap L)$ is μ-open and $(V \cap M)$ is α-H-closed set. Hence $A \cap B$ is η_μ-H-set.

Theorem 2.35. *For a subset* A *of a hereditary generalized topological space* (X, μ, H) *the following are equivalent :*

1. A *is* η_μ-H*-set*
2. $A = U \cap c_{\alpha\mathrm{H}}(A)$*, for some* μ*-open set* U*.*

Proof. (1) $\Rightarrow$ (2). Let A is η_μ-H-set. Then $A = U \cap V$, where U is μ-open V is α-H-closed. So $A \subset U$ and $A \subset V$. Which implies $c_{\alpha\mathrm{H}}(A) \subset \alpha\mathrm{H}c_\mu(V)$. Therefore, $A \subset U \cap c_{\alpha\mathrm{H}}(A) \subset U \cap \alpha\mathrm{H}c_\mu(V) = U \cap V = A$. Hence $A = U \cap c_{\alpha\mathrm{H}}(A)$.
(2) $\Rightarrow$ (1). Let $A = U \cap c_{\alpha\mathrm{H}}(A)$, for some μ-open set U. Here $c_{\alpha\mathrm{H}}(A)$ is α-H-closed. Hence A is η_μ-H-set.

Theorem 2.36. *In a hereditary generalized topologicalspace* (X, μ, H)*, the following hold:*

1. *Every* σ-H*-open is* $s\mathrm{A}_{\mathrm{H}}$*-set.*
2. *Every* H-R*-closed set is* $s\mathrm{A}_{\mathrm{H}}$*-set.*

Proof. Obvious.

Theorem 2.37. *For a subset* A *of a hereditary generalized topological space*

(X, μ, H), *the following are equivalent:*

1. *A is α-H-closed*

2. *A is αg_μ-H-closed set and η_μ-H-set.*

Proof. (1) $\Rightarrow$ (2). This is obvious.

(2) $\Rightarrow$ (1). Let A is αg_μ-H-closed set and η_μ-H-set. Since A is η_μ-H-set, then $A = U \cap c_{\alpha\mathrm{H}}(A)$, where U is μ-open in (X, μ, H). So $A \subseteq U$ and since A is αg_μ-H-closed, then $c_{\alpha\mathrm{H}} \subseteq U$. Therefore $c_{\alpha\mathrm{H}} \subseteq U \cap c_{\alpha\mathrm{H}} = A$. Hence A is α-H-closed set.

Theorem 2.38. *Let (X, μ) be a quasi topological space (X, μ) with hereditary class* H. *Then Every $s\mathrm{A}_\mathrm{H}$-set is δ-H-open.*

Proof. Let A be $s\mathrm{A}_\mathrm{H}$-set. Then $A = U \cap V$, where U is σ-H-open and V is H-R-closed. Since U is σ-H-open, $U \subset c^*_\mu i_\mu(U)$. Now $A \subset U \subset c^*_\mu i_\mu(U) \Rightarrow i_\mu c^*_\mu(A) \subset c^*_\mu i_\mu(U)$. Since V is H-R-closed, which implies $A \subset V = c^*_\mu i_\mu(V) \Rightarrow i_\mu c^*_\mu \subset i_\mu(V)$. Thus $i_\mu c^*_\mu(A) \subset c^*_\mu i_\mu(U) \cap i_\mu(V) \subset c^*_\mu(i_\mu(U) \cap i_\mu(V)) \subset c^*_\mu[i_\mu(U \cap V)] = c^*_\mu i_\mu(A)$. Hence A is δ-H-open.

Theorem 2.39. *For a subset A of a quasi topological space (X, μ) with hereditaryclass* H *the following are equivalent:*

1. *A is σ-H-open*
2. *A is strong β-H-open and $s\mathrm{A}_\mathrm{H}$-set*
3. *A is strong β-H-open and δ-H-open*

Proof. (1) $\Rightarrow$ (2). Let A is σ-H-open. Then $A \subset c^*_\mu i_\mu(A) \Rightarrow c^*_\mu(A) \subset c^*_\mu c^*_\mu i_\mu(A) = c^*_\mu i_\mu(A) \Rightarrow i_\mu c^*_\mu(A) \subset i_\mu c^*_\mu i_\mu(A) \subset c^*_\mu i_\mu(A)$. So $i_\mu c^*_\mu(A) \subset c^*_\mu i_\mu(A)$. Hence A is strong β-H-open and $s\mathrm{A}_\mathrm{H}$-set by Theorem (4.1.14.)

(2) $\Rightarrow$ (3). Let A is strong β-H-open and $s\mathrm{A}_\mathrm{H}$-set. Then A is strong β-H-open and δ-H-open by Theorem (4.1.16).

(3) $\Rightarrow$ (1) Let A is strong β-H-open and δ-H-open. Then $A \subset c^*_\mu i_\mu c^*_\mu(A)$ and $i_\mu c^*_\mu(A) \subset c^*_\mu i_\mu(A)$. Now $A \subset c^*_\mu i_\mu c^*_\mu(A) \subset c^*_\mu c^*_\mu i_\mu(A) = c^*_\mu i_\mu(A)$, which implies $\subset c^*_\mu i_\mu(A)$. Hence A is σ-H-open.

3 $(\alpha g_\mathrm{H}, \lambda)$-continuity

Definition 3.1. *A function $f : (X, \mu, \mathrm{H}) \to (Y, \lambda)$ is said to*

1. *$(\alpha_{\mathrm{H}},\lambda)$-continuous, if $f^{-1}(V)$ is a α-H-open in (X,μ,H) for each $V\in\lambda$.*
2. *$(\sigma_{\mathrm{H}},\lambda)$-continuous, if $f^{-1}(V)$ is a σ-H-open in (X,μ,H) for each $V\in\lambda$.*
3. *$(\pi_{\mathrm{H}},\lambda)$-continuous, if $f^{-1}(V)$ is a π-H-open in (X,μ,H) for each $V\in\lambda$.*
4. *$(\alpha g_{\mathrm{H}},\lambda)$-continuous, if $f^{-1}(V)$ is a αg_{μ}-H-open in (X,μ,H) for each $V\in\lambda$.*
5. *$(\eta_{\mathrm{H}},\lambda)$-continuous, if $f^{-1}(V)$ is a α-H-open in (X,μ,H) for each $V\in\lambda$.*
6. *(R_{H},λ)-continuous if $f^{-1}(V)$ is a H-R-open in (X,μ,H) for each $V\in\lambda$.*
7. *$(s\mathrm{A}_{\mathrm{H}},\lambda)$-continuous, if $f^{-1}(V)$ is a α-H-open in (X,μ,H) for each $V\in\lambda$.*

Theorem 3.2. *For a function $f:(X,\mu,\mathrm{H})\to(Y,\lambda)$, the following hold:*

1. *Every $(\alpha_{\mathrm{H}},\lambda)$-continuous function is $(\alpha g_{\mathrm{H}},\lambda)$-continuous.*
2. *Every $(\alpha g_{\mathrm{H}},\lambda)$-continuous function is $(\alpha g_{\mu},\lambda)$-continuous.*

Proof. (1). Let $f:(X,\mu,\mathrm{H})\to(Y,\lambda)$ is $(\alpha_{\mathrm{H}},\lambda)$-continuous function. Now$f^{-1}(V)$ is α - H -open for each $V\in\lambda$. Since $f^{-1}(V)$ is αg_{μ} - H -open. Hence f is $(\alpha g_{\mathrm{H}},\lambda)$-continuous.
(2). Let f is $(\alpha g_{\mathrm{H}},\lambda)$-continuous, which implies $f^{-1}(V)$ is αg_{μ}-H-open for each $V\in\lambda$. Now $f^{-1}(V)$ is αg_{μ}-open. Hence f is $(\alpha g_{\mu},\lambda)$-continuous.

Example 3.3. *Let $X=\{a,b,c,d,e\}$, $\mu=\{\emptyset,\{a\},\{b\},\{a,b\},\{b,d,e\},\{a,b,d,e\},\{a,c,d,e\},X\}$, $\mathrm{H}=\{\emptyset,\{c\}\}$, $Y=\{p,q,,r,s\}$ and $\lambda=\{\emptyset,\{p\},\{q\},\{p,q\},\{q,s,t\},\{p,q,s,t\},\{p,r,s,r\},Y\}$. Let the function $f:(X,\mu,\mathrm{H})\to(Y,\lambda)$ is definedby $f(a)=p$, $f(b)=q$, $f(c)=r$, $f(d)=s$, $f(e)=r$. Then the function f is $(\alpha g_{\mathrm{H}},\lambda)$-continuous but not $(\alpha_{\mathrm{H}},\lambda)$-continuous.*

Example 3.4. *Let $X=\{a,b,c,d,e\}$, $\mu=\{\emptyset,\{a\},\{b\},\{a,b\},\{b,d,e\},\{a,b,d,e\},\{a,c,d,e\},X\}$, $\mathrm{H}=\{\emptyset,\{c\}\}$, $Y=\{p,q,,r,s\}$ and $\lambda=\{\emptyset,\{p\},\{q\},\{p,q\},\{q,s,t\},\{p,q,s,t\},\{p,r,s,r\},Y\}$. Let the function $f:(X,\mu,\mathrm{H})\to(Y,\lambda)$ is defined by $f(a)=p$, $f(b)=q$, $f(c)=r$, $f(d)=s,f(e)=r$. Then the function f is $(\alpha g_{\mathrm{H}},\lambda)$-continuous but not $(\alpha g_{\mu},\lambda)$-continuous.*

Theorem 3.5. *For a function $f:(X,\mu,\mathrm{H})\to(Y,\lambda)$, the following hold:*

1. *Every $(\sigma_{\mathrm{H}},\lambda)$-continuous function is $(s\mathrm{A}_{\mathrm{H}},\lambda)$-continuous.*
2. *Every (R_{H},λ)-continuous function is $(s\mathrm{A}_{\mathrm{H}},\lambda)$-continuous.*

Proof. Obvious.

Example 3.6. *Let* $X = \{1, 2, 3\}$, $\mu = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$, $\mathrm{H} = \{\emptyset, \{3\}\}$ *and* $\lambda = \{\emptyset, \{3\}, X\}$. *Then the identity function* $f : (X, \mu, \mathrm{H})$ $\rightarrow (Y, \lambda)$ *is* $(s\mathrm{A}_{\mathrm{H}}, \lambda)$*-continuous but not* $(\sigma_{\mathrm{H}}, \lambda)$*-continuou.*

Example 3.7. *Let* $X = \{1, 2, 3, 4\}$, $\mu = \emptyset, \{2\}, \{1, 4\}, \{1, 2, 4\}, X\}$, $\mathrm{H} = \{\emptyset, \{1\}\}$ *and* $\lambda = \{\emptyset, \{1, 2, 4\}, X\}$. *Then the identity function* $f : (X, \mu, \mathrm{H}) \rightarrow (Y, \lambda)$ *is* $(s\mathrm{A}_{\mathrm{H}}, \lambda)$*-continuous but not* $(R_{\mathrm{H}}, \lambda)$*-continuous.*

4 Decomposition of $(\alpha_{\mathrm{H}}, \lambda)$-continuity

Definition 4.1. *A function* $f : (X, \mu, \mathrm{H}) \rightarrow (Y, \lambda)$ *is said to*

1. *strong* $(\beta_{\mathrm{H}}, \lambda)$*-continuous, if* $f^{-1}(V)$ *is a strong* β-H*-open in* (X, μ, H) *foreach* $V \in \lambda$.
2. $(\delta_{\mathrm{H}}, \lambda)$*-continuous, if* $f^{-1}(V)$ *is a* δ-H*-open in* (X, μ, H) *for each* $V \in \lambda$.

Theorem 4.2. *For a function* $f : (X, \mu, \mathrm{H}) \rightarrow (Y, \lambda)$, *the following hold:*

1. *f is* $(\alpha_{\mathrm{H}}, \lambda)$*-contiuity*
2. *f is* $(\alpha g_{\mathrm{H}}, \lambda)$*-continuity and* $(\eta_{\mathrm{H}}, \lambda)$*-continuity.*

Proof. This is obvious from Theorem 2.37.

Theorem 4.3. *For a function* $f : (X, \mu, \mathrm{H}) \rightarrow (Y, \lambda)$, *the following hold:*

1. *f is* $(\sigma_{\mathrm{H}}, \lambda)$*-contiuity*
2. *f is strong* $(\beta_{\mathrm{H}}, \lambda)$*-continuity and* $(s\mathrm{A}_{\mathrm{H}}, \lambda)$*-continuity.*
3. *f is strong* $(\beta_{\mathrm{H}}, \lambda)$*-continuity and* $(\delta_{\mathrm{H}}, \lambda)$*-continuity.*

Proof. This is obvious from Theorem 2.39.

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