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Fuzzy Integrated Mathematical Model for Optimized Inventory Management in an Industrial Environment

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Abstract

Now-a-days, the growth of business is very fast and their impact on economy is becoming bigger. To manage the inventory effectively and efficiently is always a challenge for any business organization. Keeping inventory is certainly nothing new. Every enterprise needs inventory for smooth running of its activities. It serves as a link between production and distribution processes. There is, generally, a time lag between the recognition of a need and its fulfillment. The greater the time – lag, the higher will be the requirements for inventory. Mathematical models development and determining the optimal inventory control strategy is related with this problem. Features of inventory management models are that the resulting optimal solutions can be implemented in a fast changing situation where, for example, the conditions are changed daily. Objective of this paper is to investigate the effect of uncertainty in associated key parameters and to propose managerial insights to manage inventory systems within uncertain framework. This paper presents fundamental inventory model integrated with fuzzy system for inventory management..

Keywords: Inventory, Uncertainty, Control model, Inventory management system

1. Introduction

Inventories [1] are raw materials, work-in-process goods and completely finished goods that are considered to be the portion of business's assets that are ready or will be ready for sale. Formulating a suitable inventory model [2] is one of the major concerns for an industry. The earliest scientific inventory management researches date back to the second decade of the past century, but the interest in this scientific area [3] is still great. Again considering the reliability of any process is an important feature in the research activities. Values of some factors are very hard to define or almost unreal. There is a need for new and effective methods [4] for modeling systems associated with inventory management, in the face of uncertainty. Uncertainty exists regarding the control object, as the process of obtaining the necessary information about the object is not always possible. The solution of such complex tasks requires the use of systems analysis, development of a systematic approach [5] to the problem of management in general. Inventory models are distinguished by the assumptions made about the key variables: demand, the cost structure, physical characteristics of the system [6]. These assumptions may not suit to the real environment. There is a great deal of uncertainty and variability.

2. Inventory Management Systems

Scientific inventory management system is adopted by most of the companies and industries [7]. It comprises the following steps:

1. Formulate a mathematical model describing the behavior of the inventory system.

2. Seek an optimal inventory policy [8] with respect to this model.

3. Use a computerized information processing system to maintain a record of the current inventory levels.

4. Using this record of current inventory levels, apply the optimal inventory policy to signal when and how much to replenish inventory [9].

3. Different Environment

The parameters, like inventory cost (viz., unit cost, holding cost, set-upcost, shortage cost, transportation cost, advertisement cost, etc.), demand, lead time, quantity, available resources, goals [10,11,12], etc., involved in the inventory system may be deterministic (crisp/ precise) or some of these may be non-deterministic [13] (i.e. stochastic or imprecise or both stochastic and imprecise). So the environments in which inventory models are developed can be classified as shown in the following figure.

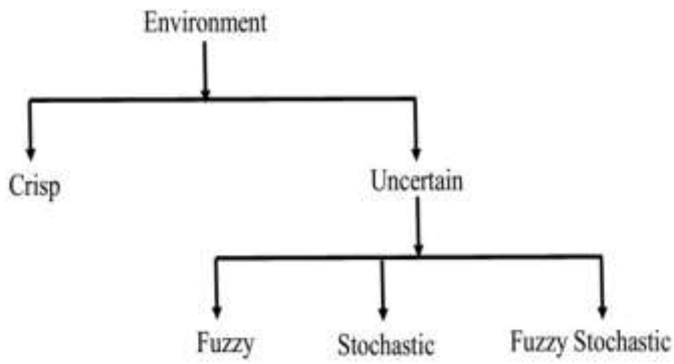

Figure: 1 Inventory Environments

Crisp Environment: When all the system parameters and the resources, etc., are deterministic and precisely defined, the environment is known as crisp environment.

Stochastic Environment: In this environment, it may happen that the demand or any factor of a commodity in the society is uncertain, not precisely known, but some past data about it is available. From the available records, the probability distribution [14] of demand or any other factor of the commodity can be determined and with that distribution the inventory control [15] problem can be analyzed and solved.

Fuzzy Environment: When managements launch a new product then they have to knowledge about demand and the other factors related to the product. Then management needs to collect the demand and the others information from experts. If the experts' opinion is imprecise then demand or other factors related to the expert opinion to be taken as a fuzzy and the corresponding environment is known as Fuzzy Environment [16].

Fuzzy-Stochastic Environment: It is an environment, which is the combination of both stochastic and fuzzy environments. For example, the statement the probability of having large demand of cricket world cup ticket is 99% involves randomness and impreciseness together. Here large is fuzzy and probability [17] represents randomness.

4. Different Inventory Types

There are four main types of inventory: raw materials/components, WIP, finished goods and MRO [18]. However, some where it is recognize by only three types of inventory, leaving out MRO. Broad Classification of inventories is shown in below figure:

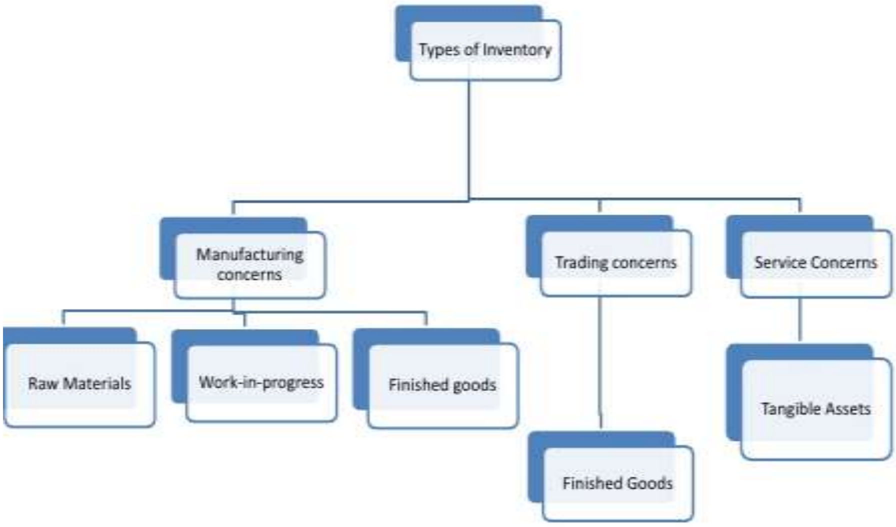

Figure: 2 Inventory Classifications

Understanding the different types of inventory is essential for making sound financial [19] and production planning choices.

5. Fuzzy Integration with Fundamental Inventory Management Model

Fuzzy logic has frequently been used to study imprecision and uncertainty [20] in inventory management systems. Fuzzy logic is often applied if only an imprecise mathematical, empirical description of a situation or a problem is available. Economic order quantity (EOQ) model [21] with fuzzy demands improves the information base by reducing uncertainty. EOQ is the order quantity that minimizes the total holding costs and ordering costs. In this model it is assumed that there is constant demand (a kg/day) for the product [22, 23]. This model is valid for constant demand for infinite time. The expected total annual cost for the classical EOQ model is given as

$$EAC\ (Q) = A\frac{D}{Q} + \frac{Q}{2}h \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where

D = Average demand/year,

A = Fixed ordering cost/order,

h = Inventory carrying costs/unit/year, and

Q = Order quantity.

It may be reformulated as

$$F_Q(D) = EAC(Q) = A\frac{\dot{D}}{Q} + \frac{Q}{2}h \quad \ldots\ldots\ldots\ldots\ldots.. \quad (2)$$

The annual demand D is replaced with fuzzy number $\dot{D}$

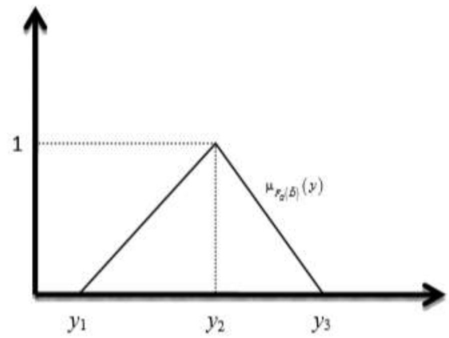

Figure: 3 Triangular fuzzy numbers $\dot{D}$

The centroid of above figure is defined ad Triangular fuzzy number $\dot{D}$ which is a fuzzy estimation of demand and is given as

$$\dot{D} = (D-\Delta_1, D, D+\Delta_2) \quad \ldots\ldots\ldots\ldots (3)$$

The fuzzy estimation of demand is given as

$$D^{*} = D + \frac{1}{3}(\Delta_2 - \Delta_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

An EOQ model with fuzzy demand and learning in fuzziness

$$\mu_{F_Q(\dot{D})}(y) = \begin{cases} \dfrac{Qy - \dfrac{Q^2 h}{2}}{A\Delta_1} - \dfrac{D-\Delta_1}{\Delta_1} & \text{if} \quad y_1 \le y \le y_2 \\ \dfrac{D+\Delta_2}{\Delta_2} + \dfrac{\dfrac{Q^2 h}{2} - Qy}{A\Delta_2} & \text{if} \quad y_2 \le y \le y_3 \\ 0 & \text{otherwise} \end{cases} \quad \ldots\ldots\ldots\ldots (5)$$

Where

$$y_1 = \frac{(D-\Delta_1)A}{Q} + \frac{Q}{2}h \quad \ldots\ldots\ldots\ldots (6)$$

$$y_2 = \frac{DA}{2} + \frac{Q}{2}h \quad \ldots\ldots\ldots\ldots (7)$$

and

$$y_3 = \frac{(D+\Delta_2)A}{Q} + \frac{Q}{2}h \quad \ldots\ldots\ldots\ldots (8)$$

An EOQ model with fuzzy demand is given as

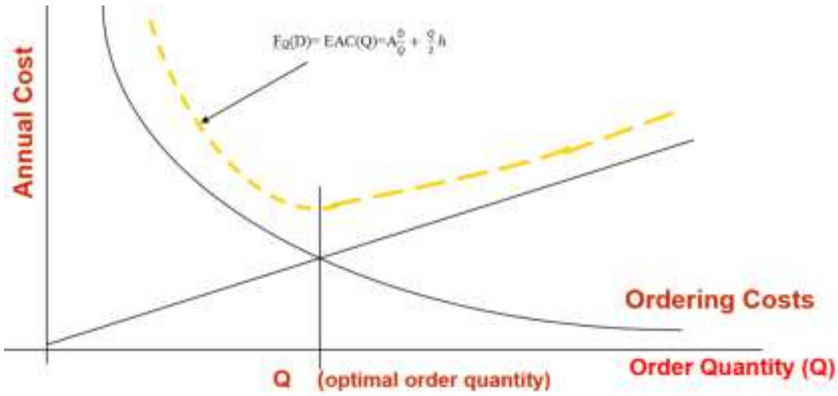

Figure: 4 EOQ model with fuzzy demand

By using calculus method, the derivative [24] of the total cost function and set the derivative (slope) equal to zero to solve for Q.

$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(Annual\ Demand)(Order\ or\ Setup\ Cost)}{Annual\ Holding\ Cost}} \quad \ldots\ldots\ldots\ldots (9)$$

Inventory Profile or sorting: It is intended for the implementation of and accounting [25] for movements and on-hand inventory as they are related to a kind of activity.

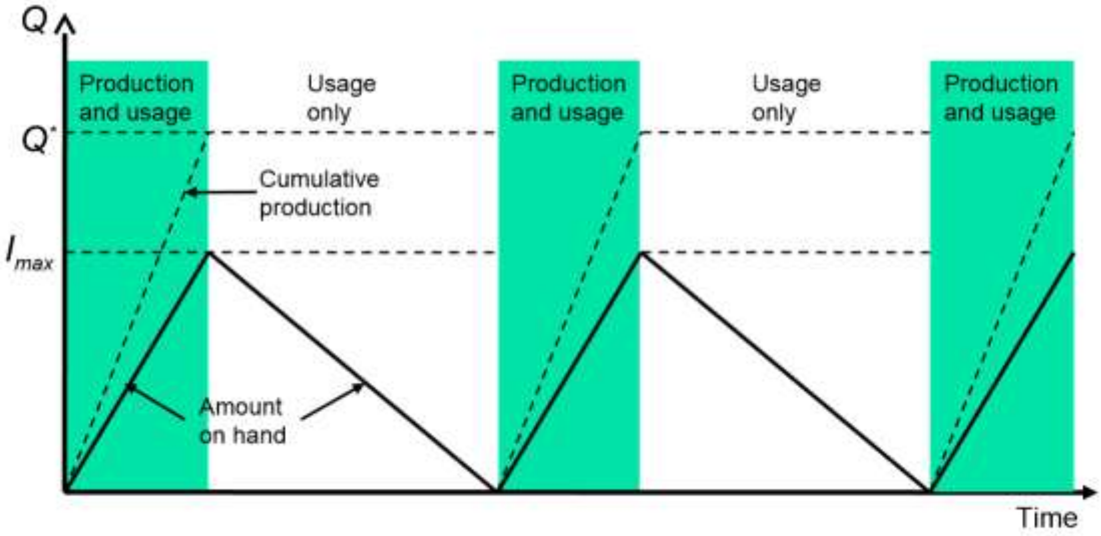

Figure: 5 Inventory Profile

It gives adjusted total cost as

$TC_{EQP}= S\frac{D}{Q} + \frac{\text{Imax}}{2} h$ ……… (10)

Maximum Inventory as

Imax = $Q| 1-\frac{d}{p}|$ ………. (11)

Adjusted order quantity as

EQP= $\sqrt{\frac{2DS}{H|\,1-\frac{d}{p}|}}$ ……… (12)

Before obtaining the optimal policy for the model, it is required to verify that the total profit/unit time.

6. Inventory Analysis

Inventory analysis [26] is the study of how product demand changes over time. This analysis helps businesses stock the right amount of goods and project how much customers will want in the future.

To perform an ABC analysis [27], group goods into three categories:

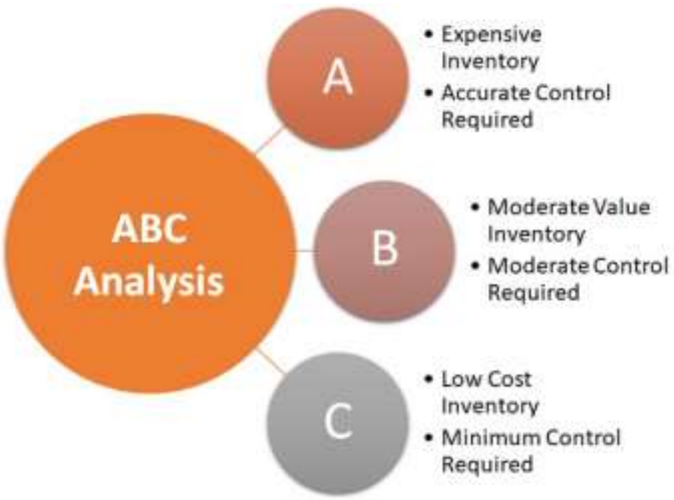

Figure: 6 ABC Inventory Analysis

ABC analysis leverages the Pareto, or 80/20, principle [28] and should reveal the 20% of your inventory that garners 80% of your profits. A company will want to focus on these items to increase sales and net profit margins [29].

Purpose of doing this ABC classification is identifying things which effect output the most. Low variation in these things can lead to high changes in output [30]. Thus these things are closely monitored thus helping organizations to increase their efficiency.

Table 1 ABC analysis classifies the inventory [31]

Type of Inventory	Type of Importance	Type of control
A Items	Very Important	Accurate records, very tight control
B Items	Little less important	Decent records, a little less tight control
C Items	Marginally important	Only essential records, light controls

Consider an inventory model with crisp parameters having the following values: demand rate D =50,000 units/year, ordering cost A=Rs.100/cycle, holding cost h=Rs.5/unit/year, screening rate x=1 unit/min (equivalently, x=175,200 units/year), screening cost d= Rs.0.5/unit, purchase cost p= Rs.25/unit, selling price of good-quality items s= Rs.50/unit, and selling price of imperfect quality items v= Rs.20/unit. The values in which the fuzzy parameters can fluctuate are respectively set as (5000, 10000), (6, 5), (0.1, 0.35), (0.0005, 0.15), (1, 2) for D, A, d, γ, h. The learning exponents are set from 0.862 (very slow learning) to 0.074 (very fast learning). The optimal policy was derived for different total number of shipments in a cycle, which adopts 5, 10, 15 and 20. The obtained results show that, comparing with the crisp model the optimal policy of the learning-fuzzy model is fairly sensitive to learning rate. When the number of orders is 5 and learning exponent shifts from 0.862 to 0.074, the optimal order quantity increases from 1438.11 to 1450.87.

The following notations are being made for developing the mathematical models:

p purchasing price of unit product

d screening cost per unit

A buyer’s ordering cost per cycle

γ fraction of defective items

Q lot size per cycle

h buyer’s holding cost per unit of time

x screening rate

D demand rate per unit of time

v unit selling price of defective items

s unit selling price of non-defectives (good) items

f(γ) probability density function of γ

N(γ,y) number of good items in each order

t screening time T cycle length

li the lower deviation value (spread) value for ith parameter i = D, h, γ, d, A

ui the upper deviation value (spread) value for ith parameter i = D, h, γ, d, A

The model with learning in fuzziness, the impact of different learning rates on the optimal order quantity and total profit/unit time are examined and plotted in below table:

Table: 2 Comparison of crisp, fuzzy and fuzzy learning model

	Crisp model		Fuzzy model		Fuzzy learning model											
	Δ_l	Δ_u	Δ_l	Δ_u	Δ_l	Δ_u	Δ_l	Δ_u	Δ_l	Δ_u	Δ_l	Δ_u	Δ_l	Δ_u
D	0	0	5000	3000	5000	3000	5000	3000	5000	3000	5000	3000	5000	3000
A	0	0	6	8	6	8	6	8	6	8	6	8	6	8
d	0	0	0.3	0.2	0.3	0.2	0.3	0.2	0.3	0.2	0.3	0.2	0.3	0.2
γ	0	0	0.01	0.03	0.01	0.03	0.01	0.03	0.01	0.03	0.01	0.03	0.01	0.03
h	0	0	1	2	1	2	1	2	1	2	1	2	1	2
i	NA		NA		5		5		5		5		5	
β	NA		NA		0.862		0.621		0.415		0.234		0.074	
Q^*	1,434.61		1,414.66		1,429.06		1,426.58		1423.65		1,420.30		1,418.17	
$\tau(T\check{P}U(Q))$	1,212,235		1,192,435.13		1,199,290.56		1,198,256.81		1,196,968.71		1,195,389.97		119,4321.97	
i	NA		NA		10		10		10		10		10	
β	NA		NA		0.86		0.737		0.515		0.234		0.074	
Q^*	1,434.61		1,414.66		1,431.49		1,429.28		1,426.24		1,422.26		1,417.41	
$\tau(T\check{P}U(Q))$	1,212,235		1,192,435.13		1,200,255.54		1,199,380.64		1,198,111.14		1,196,329.95		1,193,926.07	
i	NA		NA		15		15		15		15		15	
β	NA		NA		0.86		0.737		0.515		0.234		0.074	
Q^*	1434.61		1414.66		1,432.39		1,430.43		1,427.47		1,423.30		1,417.86	
$\tau(T\check{P}U(Q))$	1,212,235		1,192,435.13		1,200,599.81		1,199,839.55		1,198,634.91		1,196,807.90		1,194,160.39	
i	NA		NA		20		20		20		20		20	
β	NA		NA		0.86		0.737		0.515		0.234		0.074	
Q^*	1434.61		1414.66		1432.87		1429.83		1428.24		1423.98		1418.17	
$\tau(T\check{P}U(Q))$	1,212,235		1,192,435.13		1,200,779.70		1,199,601.18		1,198,954.34		1,197,118.61		1,194,321.97	

This comparison indicates that there is slight change in learning-fuzzy model; however learning-fuzzy model generates lower profit. When the learning rate is slow decision maker should order smaller lot size to the supplier, which incur lower total profit. In contrast, faster learning results in greater lot sizes with higher total profit. Therefore, learning in fuzziness makes the buyer order more and more, which tends to increase the total profit.

7. Conclusion

A multi-echelon inventory optimization solution can drive global supply chains to peak efficiency by recommending precise inventory levels, lot sizes, replenishment plans and locations from end to end. It have been treated as stochastic processes and described by probability distributions. A probability distribution is usually derived from evidence recorded in the past. This requires a valid hypothesis that collected evidences are complete and unbiased, and that the stochastic mechanism generating the recorded date continues unchanged. There are, however, situations where all these requirements are not satisfied and, therefore, the conventional probabilistic reasoning methods are not appropriate. Existing analytical method could not cover all the variables with stochastic properties in the supply chain environment. In further research, I will develop the simulator for generating graphical output data such that decision makers can see how the Inventory management system acts over time during simulation. The model developed in this paper is fundamental model but it could be extended in several directions. Moreover comprehensive study is needed to analyze the model with the data of learning process gained from real world inventory problems

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