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Research Article

An Optimal Solution for Improved Method to Solving Piecewise Quadratic Fuzzy Transportation Problem

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Abstract

A special case of the linear programming problem is the transportation theory. It also examines the case in which goods are transported from suppliers to activities. The primary aim is to minimize the overall cost of shipment while maintaining both the manufacturing limitation and the demand criterion. A new approach for solving piecewise quadratic fuzzy transportation problems for finding an optimal solution to a transport problem is included in this article. The most attractive aspect of this approach is that it requires very simple arithmetical and logical modeling. Therefore, it is very straightforward and easy to learn and use.

Keywords: *Transportation Problem (TP); Triangular Fuzzy Number (TrFN); Piecewise Quadratic Fuzzy Number (PQFN); Piecewise Quadratic Fuzzy Transportation Problem (PQFTP);*

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Introduction

In many scientific domains, such as systems analysis and operations research, we may be Biomed to set up a type using a data, which is only approximately known, due to the dimness of perception. Fuzzy numerical values can be defined on the real line, known as fuzzy numbers, by means of fuzzy subsets. The most readily conformable theory to truth is the Fuzzy set theory, according to other mathematical theories.

In life, for the most part, data retrieved for choice making are solely especially known. Dantzig, G.B. & M.N. Thapa [5], One of the earliest and most significant implementations of linear programming problem is a transportation problem. In 1965, Zadeh [16] has delivered in the idea of a fuzzy set concept to deal with such problems. In 1987, Dubois & Prade[6] represented a fuzzy range as a fuzzy subset of the proper line. Fuzzy numbers provide us to produce the mathematical model of a linguistic variables or fuzzy environments. Abbasbandy & Hajjari[1] made regarded a novel strategy ranking trapezoidal fuzzy numbers, especially based totally on the left and perfect spreads. Venkatachalapathy.M & A. Edward Samuel[13] evaluated An choice technique for fixing fuzzy transportation hassle the usage of ranking functions. Amarpreet Kaur & Amit Kumar [3] proposed a weighted distance fuzzy number. Althada Ramesh Babu and B. Rama Bhupal Reddy [2], developed a method for solving Feasibility of Fuzzy New Method in finding Initial Basic Feasible Solution for a Fuzzy Transportation Problem. Stephan Dinagar D and Christoper raj B[12], presents a method for solving fully fuzzy transportation problem with generalized quadrilateral fuzzy numbers. Muthuperumal S, Titus P & Venkatachalapathy M [9], superior a new manner for an algorithmic approach to solve unbalanced TrFTP. Venkatachalapathy M, A.Jayaraja & A.Edward Samuel [14] developed a learn about on fixing octagonal the fuzzy numbers proposed Modified Vogel's Approximation Method, Venkatachalapathy.M, Pandiarajan.R and Ganesh kumar [15] presented a method for solving generalized quadratic fuzzy number.which is an algorithm for acquiring the most appropriate solution. Bilqis Amaliah, Chastine Fatichah[4], presents one of the key steps towards achieving an optimal solution for TP is the IBFS. A new method named TOCM-MT for calculating TP's IBFS was developed. TOCM-MT is encoded using the language of C++ programming. A cumulative cost that is equivalent or closer to the optimum solution can be obtained by TOCM-MT. Nirbhay Mathur and Pankaj Kumar Srivastava[11] presents wraps an innovative approach to optimize transportation problems through generalized trapezoidal numbers in a fuzzy environment. A.Nagoor Gani, S. Abbas[10], A new method for solving intuitionistic fuzzy transportation problem proposed the ranking of fuzzy numbers and algebraic operations. The challenge of rating fuzzy numbers suitable for LP concerns with fuzzy constraints and fuzzy coefficients was also presented. More precisely, to demonstrate that reflexive fuzzy orders are constraints of the induced fuzzy order on certain special classes of fuzzy numbers.

In real life, unpredictable scenarios emerge, provided, among others, the confusion concerned with making decisions and a lack of intelligence. In some cases, it is no longer usually viable to get applicable and unique information for the fee parameter, and such imprecise facts is now not represented with the aid of a random variable chosen from a chance distribution. Zimmermann[17] confirmed that options received through the fuzzy linear programming technique are continually efficient. This paper focuses on enhancing the given balanced generalized quadratic fuzzy transportation problem to reap an preliminary primary viable solution.

In section 2, provide preliminaries of PQFN. Section 3 deals with the algorithms for optimal solutions of PQFTP. In section 4, deals with the illustrate example of PQFN to obtained optimal solution. In section 5, results and comparison of existing methods. In section 6, conclusion of the proposed method.

Preliminaries

Formulation of Generalized Quadratic Fuzzy Transportation Model:

Let ϕ be the piecewise quadratic fuzzy total distribution costs and x_{ij} (i = 1, 2, . . . , m; j = 1, 2, . . . , n) be the number of units to be distributed from source i to destination j, the linear programming formulation of this problem is

Minimize $\phi = \sum_{\mathrm{i}=1}^{m} \sum_{\mathrm{j}=1}^{n} \tilde{a}_{ij} x_{ij}$

subject to

$$\sum_{\mathrm{j}=1}^{n} x_{ij} \leq \mathrm{PQF}\,\tilde{\mathrm{S}}_{\mathrm{i}}\,,\ \mathrm{i} = 1,2,\ldots,\mathrm{m}$$

$$\sum_{\mathrm{i}=1}^{m} x_{ij} \geq \mathrm{PQF}\,\tilde{\mathrm{D}}_{\mathrm{j}}\,,\ \mathrm{j} = 1,2,\ldots,\mathrm{n}$$

$$x_{ij} \geq 0\,,\ \mathrm{i} = 1,2,\ldots,\mathrm{m},\ \ \mathrm{j} = 1,2,\ldots,\mathrm{n}$$

The fuzzy cost $\tilde{a}_{ij}$ are all non-negative. If $\sum_{i=1}^{m} PQF\,\tilde{S}_i = \sum_{j=1}^{n} PQF\,\tilde{D}_j$, it is a balanced piecewise quadratic fuzzy transportation problem .If this condition is not met, a dummy origin or destination is generally introduced to make the problem balanced. The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problems.

Definition : Fuzzy Set

The fuzzy set $\tilde{A}$ is defind by $\tilde{A}=\{(x,\mu_{\tilde{A}}(x): x\in A \ and \ \ \mu_{\tilde{A}}(x)\in[0,1]\}$, $\mu_{\tilde{A}}(x)$ is called the membership function.

Definition: Fuzzy Number

A real piecewise quadratic fuzzy number $\tilde{A}=(s_1,s_2,s_3,s_4,s_5)$ is a fuzzy subset from the real line $\Re$ with the membership function $\mu_{\tilde{A}}(x)$ satisfying the following conditions

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\Re$ to the closed interval[0,1].

(ii) $\mu_{\tilde{A}}(x)=0$ for every $\tilde{A}\in(-\infty,s_1]$.

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing and continuous on [s_1,s_2].

(iv) $\mu_{\tilde{A}}(x)$ is strictly increasing and continuous on [s_2,s_3].

(v) $\mu_{\tilde{A}}(x)$ is a strictly decreasing and continuous on [s_3,s_4].

(vi) $\mu_{\tilde{A}}(x)$ is a strictly decreasing and continuous on [s_4,s_5].

(vii) $\mu_{\tilde{A}}(x)=0$ for every $\tilde{A}\in[s_5,\infty)$.

The membership values assigned to distinct values are represented diagrammatically below $\left\{\frac{0}{s_1},\frac{0.5}{s_2},\frac{1}{s_3},\frac{0.5}{s_4},\frac{0}{s_5}\right\}$.

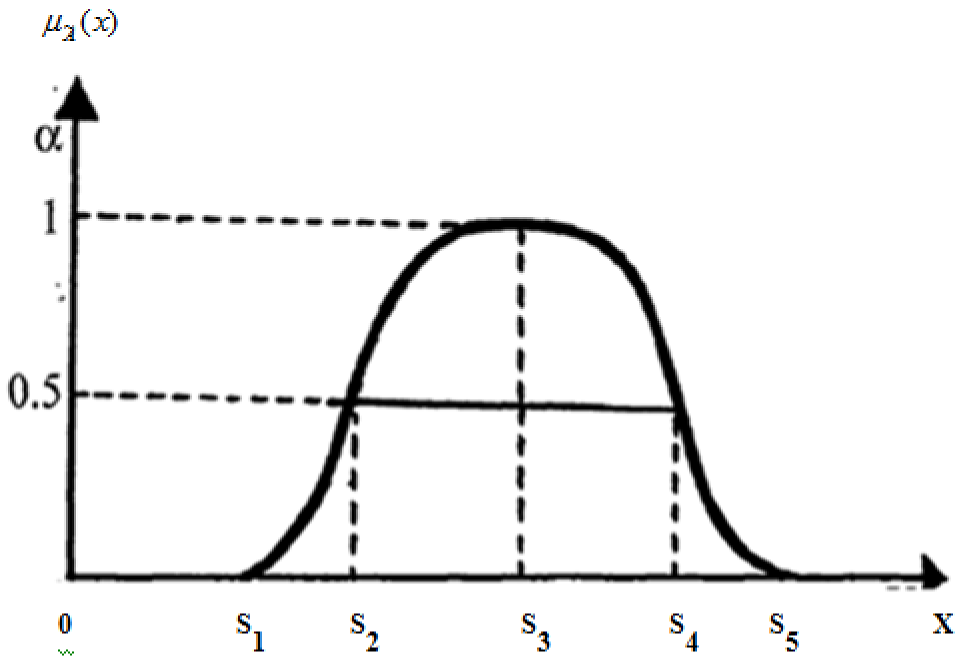

Figure 1 Piecewise Quadratic Fuzzy Number (PQFN)

DEFINITION: Piecewise Quadratic Fuzzy Numbers (PQFN):

A PQFN $\tilde{A}$ is basically a fuzzy number denoted as $\tilde{A}=(s_1,s_2,s_3,s_4,s_5)$ and is denoted by the membership function as

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{1}{2(s_2-s_1)^2}(x-s_1)^2 & for\ s_1\le x\le s_2 \\ \frac{1}{2(s_3-s_2)^2}(x-s_3)^2+1 & for\ s_2\le x\le s_3 \\ \frac{1}{2(s_4-s_3)^2}(x-s_3)^2+1 & for\ s_3\le x\le s_4 \\ \frac{1}{2(s_5-s_4)^2}(x-s_5)^2 & for\ s_4\le x\le s_5 \\ 0 & otherwise \end{cases}$$

The PQFN is a bell shaped curve symmetric about the line $x=s_3$,possess a supporting interval $[s_1,s_5]$.Moreover, $s_3=\frac{1}{2}(s_1+s_5)$ and $s_3-s_2=s_4-s_3$. The α -cut for $\alpha=\frac{1}{2}$ between the points (s_2,s_4) and they are called cross over points. The interval of confidence at level α is given to be $A_\alpha=\{s_1+(s_3-s_1)\alpha, s_5-(s_5-(s_5-s_3)\alpha\}$.

Algebraic Operations:

In this part, algebraic operations of two piecewise QFN, this declared on the accepted set of absolute numbers.

Let $\tilde{A}=(s_1,s_2,s_3,s_4,s_5)$ and $\tilde{B}=(t_1,t_2,t_3,t_4,t_5)$ are two PQFN

(i). Addition : $\tilde{A}+\tilde{B}=(s_1+t_1,s_2+t_2,s_3+t_3,s_4+t_4,s_5+t_5)$

(ii). Subtraction : $\tilde{A}-\tilde{B}=(s_1-t_5,s_2-t_4,s_3-t_3,s_4-t_2,s_5-t_1)$

(iii). Scalar Multiplication:
$$\lambda\tilde{A}=(\lambda s_1,\lambda s_2,\lambda s_3,\lambda s_4,\lambda s_5)\ for\ \lambda>0$$
$$\lambda\tilde{A}=(\lambda s_5,\lambda s_4,\lambda s_3,\lambda s_2,\lambda s_1)\ for\ \lambda<0$$

(iv).Multiplication:

$$\tilde{A}\otimes\tilde{B}=\left(\frac{1}{2}(s_5t_1+s_1t_5),\frac{1}{2}(s_4t_2+s_2t_4),s_3t_3,\frac{1}{2}(s_2t_2+s_4t_4),\frac{1}{2}(s_1t_1+s_5t_5)\right)$$

Ranking of Piecewise Quadratic Fuzzy Numbers:

If $\tilde{A}=(s_1,s_2,s_3,s_4,s_5)$ is a PQFN then its associated ordinary crisp number is given by $R(\tilde{A})=\frac{s_1+s_2+s_3+s_4+s_5}{5}$.

Proposed Algorithms of Piecewise Quadratic Fuzzy Transportation Problems:

S1: Develop the transportation table from given PQFTransportation Problem. Construct the cost matrix of the given TP and check whether matrix is balanced or unbalanced. If it is unbalanced, i.e. total demand is not equal to total supply then introduce dummy row or column and assign that quantity to either demand or supply side, which equals total demand and total supply. Put zero cost in the individual cells of the dummy row or dummy column.

S2: Subtract each row entries of the transportation table from the respective row minimum element. Similarly, subtract each column entries from respective column minimum element.

S3: The reduced cost matrix will have at least one zero entries in each row and in each column. Proceed with zero entries occurring in the row of cost matrix. Assume this zero occurs in $(i,j)^{th}$ cell, count the remaining zeros in the i^{th} row and j^{th} column. Select the next row, which have a zero cell entry and proceed in the same manner until all zeros are covered.

S4: Find a zero entry cell, which have the minimum zero count in the ith row and jth column and assign the maximum amount satisfying the supply and demand sides. If tie occurs for minimum zero count then choose that zero cell for which total sum of all the elements of respective row and column is maximum.

S5: Eliminate the rows or columns for which demand or supply is already exhausted before carrying out further calculation.

S6: Check if the reduced cost matrix has at least one zero in each row and in each column. If not, start S2 again, otherwise move to next step.

S7: If the demand and the supply remain unfilled, start S3 to S6 again until all the demand and all the supply are exhausted.

An illustrate Example

Consider the following Piecewise quadratic fuzzy transportation problem. To obtain the optimal solution.

Table 1Balanced Piecewise quadratic fuzzy transportation problem

	Worker 1	Worker 2	Worker 3	Worker 4	Availability
Job 1	(8,10,11,12,14)	(10,12,13,14,16)	(14,16,17,18,20)	(11,13,14,15,17)	(247,249,250,251, 253)
Job 2	(13,15,16,17,19)	(15,17,18,19,21)	(11,13,14,15,17)	(7,9,10,11,13)	(297,299,300,301, 303)
Job 3	(18,20,21,22,24)	(21,23,24,25,27)	(10,12,13,14,16)	(7,9,10,11,13)	(397,399,400,401, 403)

Requirements	(197,199,200,201,203)	(222,224,225,226,228)	(272,274,275,276,278)	(247,249,250,251,253)	

Table 2 Row reduction of Piecewise quadratic fuzzy matrix

(-6,-2,0,2,6)	(-4,0,2,4,8)	(0,4,6,8,12)	(-3,1,3,5,9)
(0,4,6,8,12)	(2,6,8,10,14)	(-2,2,4,6,10)	(-6,-2,0,2,6)
(5,9,11,13,17)	(8,12,14,16,20)	(-3,1,3,5,9)	(-6,-2,0,2,6)

Table 3 Column reduction of Piecewise quadratic fuzzy matrix

(-12,-4,0,4,12)	(-12,-4,0,4,12)	(-9,-1,3,7,15)	(-9,-1,3,7,15)
(-6,2,6,10,18)	(-6,2,6,10,18)	(-11,-3,1,5,13)	(-12,-4,0,4,12)
(-1,7,11,15,23)	(0,8,12,16,24)	(-12,-4,0,4,12)	(-12,-4,0,4,12)

Table 4 First allotment with the reduction of Penalties

(197,199,200,201) (-12,-4,0,4,12)	(-12,-4,0,4,12)	(-9,-1,3,7,15)	(-9,-1,3,7,15)	(247,249,250,251,253) -(197,199,200,201) =(44,48,50,52,56)
(-6,2,6,10,18)	(-6,2,6,10,18)	(-11,-3,1,5,13)	(-12,-4,0,4,12)	(297,299,300,301,303)
(-1,7,11,15,23)	(0,8,12,16,24)	(-12,-4,0,4,12)	(-12,-4,0,4,12)	(397,399,400,401,403)
(197,199,200,201,203)	(222,224,225,226,228)	(272,274,275,276,278)	(247,249,250,251,253)	

Table 5 Second allotment with the reduction of Penalties

	(44,48,50,52,56) (-12,-4,0,4,12)	(-9,-1,3,7,15)	(-9,-1,3,7,15)	(44,48,50,52,56)

***	(-6,2,6,10,18)	(-11,-3,1,5,13)	(-12,-4,0,4,12)	(297,299,300,301,303)
***	(0,8,12,16,24)	(-12,-4,0,4,12)	(-12,-4,0,4,12)	(397,399,400,401,403)
***	(222,224,225,226,228) -(44,48,50,52,56) =(166,172,175,178,184)	(272,274,275,276,278)	(247,249,250,251,253)	

Repeat the process of S3 to S6, until all the demand and all the supply are exhausted.

Table 6 Piecewise quadratic fuzzy transportation table is allocated as follows

(197,199,200,201) (8,10,11,12,14)	**(44,48,50,52,56)** (10,12,13,14,16)	(14,16,17,18,20)	(11,13,14,15,17)	(247,249,250,251,253)
(13,15,16,17,19)	**(163,171,175,179,187)** (15,17,18,19,21)	(11,13,14,15,17)	**(116,122,125,128,134)** (7,9,10,11,13)	(297,299,300,301,303)
(18,20,21,22,24)	(21,23,24,25,27)	**(272,274,275,276,278)** (10,12,13,14,16)	**(119,123,125,127,131)** (7,9,10,11,13)	(397,399,400,401,403)
(197,199,200,201,203)	(222,224,225,226,228)	(272,274,275,276,278)	(247,249,250,251,253)	

The overall piecewise quadratic fuzzy optimal solution is

=**(197,199,200,201)** (8,10,11,12,14)+ **(44,48,50,52,56)** (10,12,13,14,16)

+ **(163,171,175,179,187)** (15,17,18,19,21)+ **116,122,125,128,134)** (7,9,10,11,13)

+ **(272,274,275,276,278)** (10,12, 13,14,16)+ **(119,123,125,127,131)** (7,9,10,11,13)

=(11958,12062,12075,12088,12192)

The ranking of piecewise quadratic fuzzy transportation cost is 12075.

Results Comparison

The proposed methods piecewise quadratic fuzzy provides an FIBFS and optimal solutions that give better results as follows

Table 7 Comparison table of IBFS and Optimal Solutions

Methods	NWCM	LCM	VAM	MODI	Proposed Method
Defuzzified IBFS/Optimal Solution	**12200**	**12825**	**12075**	**12075**	**12075**

Conclusion

In this paper, a proposed method for solving PQFTP provides an optimal solution with less number of iterations and make easy to understand. So it will be helpful for decision makers who are dealing with PQFTP. Our Proposed method to compare the existing methods that yield the same optimal solutions.

Compliance with ethical standards

Conflicts of Interest:

The authors declare that there are no conflicts of interest regarding the publication of this paper. All authors have equal contributions. All authors read and approved final manuscript. The authors would like to thank to the editor and anonymous referees.This research is not supported under any funding.

Research involving human participants and/or animal

This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent

All referred study is highlighted in the Literature Review.

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