Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 13, Issue 1, January 2022: 1265-1272

Intuitionistic Fuzzy Graph Matrices Using Intuitionistic Fuzzy Graph Operations

Dr. B. Fathima Kani
Assistant Professor, Department of Mathematics ,Dr.Zakir Husain College,Ilayangudi,Sivaganga-630702,Tamil Nadu, India. Mail Id:kanidrzhc@gmail.com

Abstract

In this journal three operations αlpha product, βeta product and γamma product on IFGs are defined. And ,we discussed the theorems related to 3 operations. Every Fuzzy Graph transformed to Fuzzy Matrix. Likewise every Intuitionistic Fuzzy Graph transformed to an Intuitionistic Fuzzy Matrix. Here we determined the algebraic results of αlpha, βeta, γamma , Product Intuitionistic Fuzzy Matrices like singular, invertible and diagonalizable .

Keywords: αlpha Product ; βeta Product; γamma Product; IFG; IFM.

1. Introduction

IFM used in networking, cluster analysis, medical diagnosis. αlpha, βeta and γamma PFGs were introduced by **Fathima Kani.B , Nagoor Gani.A(2014)**. Further they defined its matrix representation of 3 FG operations**Fathima Kani.B , Nagoor Gani.A(2019)** . Here we define the matrix formation of αlpha, βeta and γamma PIFG operation .This journal is organised as follows. In sec. 2,definitions of αlpha,βeta and γamma PIFGs are discussed and properties related to SIFGs is discussed. In Sec. 3 IFMs of αlpha, βeta and γamma product are defined & the theorems related to singular, invertible and diagonalizable are discussed with suitable examples. Here FG-Fuzzy Graph, FM-Fuzzy Matrix, IFG-Intuitionistic Fuzzy Graph, IFM-Intuitionistic Fuzzy Matrix, SIFG-Strong Intuitionistic Fuzzy Graph, PIFG-Product Intuitionistic Fuzzy Graph.

2.αlpha,βeta and γamma PIFGs

Definition 2.1

The α –PIFG of $G'=(X',Y',\delta',\varepsilon')$ and $G''=(X'',Y'',\delta'',\varepsilon''),X'\cap X''=\emptyset$ is IFG,

$G'\times G''=(X,Y,\delta'\times\delta'',\varepsilon'\times\varepsilon'')$, $X=X'\times X''$,

$$\mathrm{Y}=Y'\times Y''=\left\{\left((\mathrm{u}_1,\mathrm{u}_2),(\mathrm{v}_1,\mathrm{v}_2)\right)\left|\begin{matrix}\mathrm{u}_1=\mathrm{v}_1,\mathrm{u}_2\mathrm{v}_2\in Y''(\mathrm{Or})\\ \mathrm{u}_2=\mathrm{v}_2,\mathrm{u}_1\mathrm{v}_1\in Y'(\mathrm{Or})\\ \mathrm{u}_1\mathrm{v}_1\in Y',\mathrm{u}_2\mathrm{v}_2\notin Y''(\mathrm{Or})\\ \mathrm{u}_1\mathrm{v}_1\notin Y',\mathrm{u}_2\mathrm{v}_2\in Y''\end{matrix}\right.\right\},:$$

$$\left(\delta_1{}'\times\delta_1{}''\right)(u,v)=\delta_1{}'(u)\wedge\delta_1{}''(v)$$

$$\left(\delta_2{}'\times\delta_2{}''\right)(u,v)=\delta_2{}'(u)\vee\delta_2{}''(v)$$

$$(\varepsilon_1{}' \times \varepsilon_1{}'')\big((u_1,u_2),(v_1,v_2)\big) = \begin{Bmatrix} \delta_1{}'(u_1)\wedge\varepsilon_1{}''(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in Y'' \\ \delta_1{}''(u_2)\wedge\varepsilon_1{}'(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in Y' \\ \delta_1{}''(u_2)\wedge\delta_1{}''(v_2)\wedge\varepsilon_1{}'(u_1v_1), & \text{if } u_1v_1 \in Y', u_2v_2 \notin Y'' \\ \delta_1{}'(u_1)\wedge\delta_1{}'(v_1)\wedge\varepsilon_1{}''(u_2v_2), & \text{if } u_1v_1 \notin Y', u_2v_2 \in Y'' \end{Bmatrix}$$

$$(\varepsilon_2{}' \times \varepsilon_2{}'')\big((u_1,u_2),(v_1,v_2)\big) = \begin{Bmatrix} \delta_2{}'(u_1)\vee\varepsilon_2{}''(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in Y'' \\ \delta_2{}''(u_2)\vee\varepsilon_2{}'(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in Y' \\ \delta_2{}''(u_2)\vee\delta_2{}''(v_2)\vee\varepsilon_2{}'(u_1v_1), & \text{if } u_1v_1 \in Y', u_2v_2 \notin Y'' \\ \delta_2{}'(u_1)\vee\delta_2{}'(v_1)\vee\varepsilon_2{}''(u_2v_2), & \text{if } u_1v_1 \notin Y', u_2v_2 \in Y'' \end{Bmatrix}$$

Definition 2.2

The β –PIFG of $G'=(X',Y',\delta',\varepsilon')$ and $G''=(X'',Y'',\delta'',\varepsilon'')$,$X' \cap X'' = \emptyset$ is the IFG

$G' \times G'' = (X,Y,\delta' \times \delta'', \varepsilon' \times \varepsilon'')$, $X = X' \times X''$ and

$$Y = Y' \times Y'' = \left\{ ((u_1,u_2),(v_1,v_2)) \middle| \begin{matrix} u_1 \neq v_1, u_2v_2 \in Y'' (\mathrm{Or}) \\ u_2 \neq v_2, u_1v_1 \in Y' \ (\mathrm{Or}) \\ u_1v_1 \in Y', u_2v_2 \in Y'' \end{matrix} \right\}, :$$

$$\big(\delta_1{}' \times \delta_1{}''\big)(u,v) = \delta_1{}'(u) \wedge \delta_1{}''(v)$$

$$\big(\delta_2{}' \times \delta_2{}''\big)(u,v) = \delta_2{}'(u) \vee \delta_2{}''(v)$$

$$(\varepsilon_1{}' \times \varepsilon_1{}'')\big((u_1,u_2),(v_1,v_2)\big) = \begin{Bmatrix} \delta_1{}'(u_1)\wedge\delta_1{}'(v_1)\wedge\varepsilon_1{}''(u_2v_2), & \text{if } u_1 \neq v_1, u_2v_2 \in Y'' \\ \delta_1{}'(u_2)\wedge\delta_1{}''(v_2)\wedge\varepsilon_1{}'(u_1v_1), & \text{if } u_2 \neq v_2, u_1v_1 \in Y' \\ \varepsilon_1{}'(u_1v_1)\wedge\varepsilon_1{}''(u_2v_2), & \text{if } u_1v_1 \in Y', u_2v_2 \in Y'' \end{Bmatrix}$$

$$(\varepsilon_2{}' \times \varepsilon_2{}'')\big((u_1,u_2),(v_1,v_2)\big) = \begin{Bmatrix} \delta_2{}'(u_1)\vee\delta_2{}'(v_1)\vee\varepsilon_2{}''(u_2v_2), & \text{if } u_1 \neq v_1, u_2v_2 \in Y'' \\ \delta_2{}''(u_2)\vee\delta_2{}''(v_2)\vee\varepsilon_2{}'(u_1v_1), & \text{if } u_2 \neq v_2, u_1v_1 \in Y' \\ \varepsilon_2{}'(u_1v_1)\vee\varepsilon_2{}''(u_2v_2), & \text{if } u_1v_1 \in Y', u_2v_2 \in Y'' \end{Bmatrix}$$

Definition 2.3

The γ - PIFG *of* $G'=(X',Y',\delta',\varepsilon')$ and $G''=(X'',Y'',\delta'',\varepsilon'')$,$X' \cap X'' = \emptyset$ is the IFG

$G' \times G'' = (X,Y,\delta' \times \delta'', \varepsilon' \times \varepsilon'')$, $X = X' \times X''$ and

$$Y = Y' \times Y'' = \left\{ ((u_1,u_2),(v_1,v_2)) \middle| \begin{matrix} u_1 = v_1, u_2v_2 \in Y'' (\mathrm{Or}) \\ u_2 = v_2, u_1v_1 \in Y' \ (\mathrm{Or}) \\ u_1 \neq v_1, u_2v_2 \in Y'' (\mathrm{Or}) \\ u_2 \neq v_2, u_1v_1 \in Y' \ (\mathrm{Or}) \\ u_1v_1 \in Y', u_2v_2 \in Y'' \end{matrix} \right\}, :$$

$$\big(\delta_1{}' \times \delta_1{}''\big)(u,v) = \delta_1{}'(u) \wedge \delta_1{}''(v)$$

$$\big(\delta_2{}' \times \delta_2{}''\big)(u,v) = \delta_2{}'(u) \vee \delta_2{}''(v)$$

$$(\varepsilon_1{}' \times \varepsilon_1{}'')\big((u_1,u_2),(v_1,v_2)\big) = \begin{cases} \delta_1{}'(u_1) \wedge \varepsilon_1{}''(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in Y'' \\ \delta_1{}''(u_2) \wedge \varepsilon_1{}'(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in Y' \\ \delta_1{}'(u_1) \wedge \delta_1{}'(v_1) \wedge \varepsilon_1{}''(u_2v_2), & \text{if } u_1 \neq v_1, u_2v_2 \in Y'' \\ \delta_1{}''(u_2) \wedge \delta_1{}''(v_2) \wedge \varepsilon_1{}'(u_1v_1), & \text{if } u_2 \neq v_2, u_1v_1 \in Y' \\ \varepsilon_1{}'(u_1v_1) \wedge \varepsilon_1{}''(u_2v_2), & \text{if } u_1v_1 \in Y', u_2v_2 \in Y'' \end{cases}$$

$$(\varepsilon_2{}' \times \varepsilon_2{}'')\big((u_1,u_2),(v_1,v_2)\big) = \begin{cases} \delta_2{}'(u_1) \vee \varepsilon_2{}''(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in Y'' \\ \delta_2{}''(u_2) \vee \varepsilon_2{}'(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in Y' \\ \delta_2{}'(u_1) \vee \delta_2{}'(v_1) \vee \varepsilon_2{}''(u_2v_2), & \text{if } u_1 \neq v_1, u_2v_2 \in Y'' \\ \delta_2{}''(u_2) \vee \delta_2{}''(v_2) \vee \varepsilon_2{}'(u_1v_1), & \text{if } u_2 \neq v_2, u_1v_1 \in Y' \\ \varepsilon_2{}'(u_1v_1) \vee \varepsilon_2{}''(u_2v_2), & \text{if } u_1v_1 \in Y', u_2v_2 \in Y'' \end{cases}$$

Theorem 2.4

If $G'=(X',Y',\delta',\varepsilon')$ and $G''=(X'',Y'',\delta'',\varepsilon'')$ are SIFGs ,then $G' \times G''$ is also SIFG.

Solution: Take G' and G'' as two SIFGs, then

$\varepsilon_1{}'(s,t) = \delta_1{}'(s) \wedge \delta_1{}'(t)$

$\varepsilon_2{}'(s,t) = \delta_2{}'(s) \vee \delta_2{}'(t)$

$\varepsilon_1{}''(s,t) = \delta_1{}''(s) \wedge \delta_1{}''(t)$

$\varepsilon_2{}'(s,t) = \delta_2{}''(s) \vee \delta_2{}''(t), \forall$ (u,v)$\in Y'$ and (u,v)$\in Y''$.

To show , $(\varepsilon_1{}' \times \varepsilon_1{}'')\big((s_1,s_2),(t_1,t_2)\big) = \big(\delta_1{}' \times \delta_1{}''\big)(s_1,s_2) \wedge \big(\delta_1{}' \times \delta_1{}''\big)(t_1,t_2)$

Consider α - PIFG

If $s_1 = t_1, s_2t_2 \in Y''$, $(\varepsilon_1{}' \times \varepsilon_1{}'')\big((s_1,s_2),(t_1,t_2)\big) = \delta_1{}'(s_1) \wedge \varepsilon_1{}''(s_2t_2)$

$= \delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}''(t_2)$

$= \delta_1{}'(s_1) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}''(t_2)$

$= \delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(t_2)$

$=\big(\delta_1{}' \times \delta_1{}''\big)(s_1,s_2) \wedge \big(\delta_1{}' \times \delta_1{}''\big)(t_1,t_2)$

If $s_2 = t_2, s_1t_1 \in Y'$, $(\varepsilon_1{}' \times \varepsilon_1{}'')\big((s_1,s_2),(t_1,t_2)\big) = \delta_1{}''(s_2) \wedge \varepsilon_1{}'(s_1t_1)$

$= \delta_1{}''(s_2) \wedge \delta_1{}'(s_1) \wedge \delta_1{}'(t_1)$

$=\delta_1{}''(s_2) \wedge \delta_1{}''(t_2) \wedge \delta_1{}'(s_1) \wedge \delta_1{}'(t_1)$

$=\delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(t_2)$

$=(\delta_1{}' \times \delta_1{}'')(s_1,s_2) \wedge (\delta_1{}' \times \delta_1{}'')(t_1,t_2)$

If $s_1t_1 \in Y', s_2t_2 \notin Y'', (\varepsilon_1{}' \times \varepsilon_1{}'')((s_1,s_2),(t_1,t_2)) = \delta_1{}''(s_2) \wedge \delta_1{}''(t_2) \wedge \varepsilon_1{}'(s_1t_1)$

$=\delta_1{}''(s_2) \wedge \delta_1{}''(t_2) \wedge \delta_1{}'(s_1) \wedge \delta_1{}'(t_1)$

$=\delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(t_2)$

$=(\delta_1{}' \times \delta_1{}'')(s_1,s_2) \wedge (\delta_1{}' \times \delta_1{}'')(t_1,t_2)$

if $s_1t_1 \notin Y', s_2t_2 \in Y'', (\varepsilon_1{}' \times \varepsilon_1{}'')((s_1,s_2),(t_1,t_2)) = \delta_1{}'(s_1) \wedge \delta_1{}'(t_1) \wedge \varepsilon_1{}''(s_2t_2)$

$=\delta_1{}'(s_1) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}''(t_2)$

$=\delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(t_2)$

$=(\delta_1{}' \times \delta_1{}'')(s_1,s_2) \wedge (\delta_1{}' \times \delta_1{}'')(t_1,t_2)$

It is valid for non members also .That is

$(\varepsilon_2{}' \times \varepsilon_2{}'')((s_1,s_2),(t_1,t_2)) = (\delta_2{}' \times \delta_2{}'')(s_1,s_2) \vee (\delta_2{}' \times \delta_2{}'')(t_1,t_2)$,

$$\forall\, Y = \left\{ ((s_1,s_2),(t_1,t_2)) \,\middle|\, \begin{array}{c} s_1 = t_1, s_2t_2 \in Y'' (0r) \\ s_2 = t_2, s_1t_1 \in Y' \ (0r) \\ s_1t_1 \in Y', s_2t_2 \notin Y'' (0r) \\ s_1t_1 \notin Y', s_2t_2 \in Y'' \end{array} \right\}$$

Consider β- PIFG

If $s_1 \neq t_1, s_2t_2 \in Y'', (\varepsilon_1{}' \times \varepsilon_1{}'')((s_1,s_2),(t_1,t_2)) = \delta_1{}'(s_1) \wedge \delta_1{}'(t_1) \wedge \varepsilon_1{}''(s_2t_2)$

$=\delta_1{}'(s_1) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}''(t_2)$

$=\delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(t_2)$

$=(\delta_1{}' \times \delta_1{}'')(s_1,s_2) \wedge (\delta_1{}' \times \delta_1{}'')(t_1,t_2)$

If $s_2 \neq t_2, s_1t_1 \in Y'$, $(\varepsilon_1{}' \times \varepsilon_1{}'')((s_1,s_2),(t_1,t_2)) = \delta_1{}''(s_2) \wedge \delta_1{}''(s_2) \wedge \varepsilon_1{}'(s_1t_1)$

$=\delta_1{}''(s_2) \wedge \delta_1{}''(t_2) \wedge \delta_1{}'(s_1) \wedge \delta_1{}'(t_1)$

$=\delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(t_2)$

$=\left(\delta_1{}' \times \delta_1{}''\right)(s_1, s_2) \wedge \left(\delta_1{}' \times \delta_1{}''\right)(t_1, t_2)$

If $s_1t_1 \in Y', s_2t_2 \in Y'', (\varepsilon_1{}' \times \varepsilon_1{}'')\big((s_1, s_2), (t_1, t_2)\big) = \varepsilon_1{}'(s_1t_1) \wedge \varepsilon_1{}''(s_2t_2)$

$= \delta_1{}'(s_1) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}''(t_2)$

$= \delta_1{}'(s_1) \wedge \delta_1{}''(s_2) \wedge \delta_1{}'(t_1) \wedge \delta_1{}''(t_2)$

$=\left(\delta_1{}' \times \delta_1{}''\right)(s_1, s_2) \wedge \left(\delta_1{}' \times \delta_1{}''\right)(t_1, t_2)$

This is valid for non-member of β -PIFG.

(i.e.) $(\varepsilon_2{}' \times \varepsilon_2{}'')\big((s_1, s_2), (t_1, t_2)\big) = \left(\delta_2{}' \times \delta_2{}''\right)(s_1, s_2) \vee \left(\delta_2{}' \times \delta_2{}''\right)(t_1, t_2)\ \forall$

$$Y = \left\{ ((s_1, s_2), (t_1, t_2)) \middle| \begin{matrix} s_1 \neq t_1, s_2t_2 \in Y'' (\text{Or}) \\ s_2 \neq t_2, s_1t_1 \in Y' \ (\text{Or}) \\ s_1t_1 \in Y', s_2t_2 \in Y'' \end{matrix} \right\}$$

Combination of α-P and β -P of IFG is a γ-P. Thus α -P ,β-P and γ -P of two SIFGs is again a SIFG.

3.Matrix Representation of α ,β and γ PIFGs

Definition 3.1

α-PIFM is A=$[u_iv_j]$,i=1,2,….n,j=1,2,….m , $[u_iv_j]=\begin{bmatrix} (\delta_1{}' \times \delta_1{}'', \ \delta_2{}' \times \delta_2{}''), i = j \\ (\varepsilon_1{}' \times \varepsilon_1{}'', \ \varepsilon_2{}' \times \varepsilon_2{}''), i \neq j \end{bmatrix}$,where n,m are $|X'|\&|X''|$ which belongs to $G'=(X', Y', \delta', \varepsilon')$ and $G'' = (X'', Y'', \delta'', \varepsilon'')$.

Definition 3.2

β - PIFM is B=$[u_iv_j]$,i=1,2,….n,j=1,2,….m , $[u_iv_j]=\begin{bmatrix} (\delta_1{}' \times \delta_1{}'', \ \delta_2{}' \times \delta_2{}''), i = j \\ (\varepsilon_1{}' \times \varepsilon_1{}'', \ \varepsilon_2{}' \times \varepsilon_2{}''), i \neq j \end{bmatrix}$,where n,m are $|X'|\&|X''|$ of $G'=(X', Y', \delta', \varepsilon')$ and $G'' = (X'', Y'', \delta'', \varepsilon'')$.

Definition 3.3

γ - PIFM is G=$[u_iv_j]$,i=1,2,….n,j=1,2,….m ,$[u_iv_j]=\begin{bmatrix} (\delta_1{}' \times \delta_1{}'', \ \delta_2{}' \times \delta_2{}''), i = j \\ (\varepsilon_1{}' \times \varepsilon_1{}'', \ \varepsilon_2{}' \times \varepsilon_2{}''), i \neq j \end{bmatrix}$,where n,m are $|X'|\&|X''|$ of $G'=(X', Y', \delta', \varepsilon')$ and $G'' = (X'', Y'', \delta'', \varepsilon'')$.

Theorem 3.4

Every α -P and β - P of IFMs are non- singular.

Proof: W.K.T ,an n×n matrix is non-singular ,if Ax=0 has only the zero solution. We split this matrix in-to 2 ,one is membership matrix , another one is non- membership matrix.

Consider α -PIFGs. Here M - membership matrix and NM - non-membership matrix.

```
>> M=[0.2  0.2  0.2  0; 0.2  0.2  0  0.2 ; 0.2  0  0.3  0.3 ; 0  0.2  0.3  0.3];
>> null(M)
ans =Null matrix: 4×0
>> NM=[0.4  0.7  0.6 0;0.7 0.7 0 0.7;0.6 0 0.6 0.7;0 0.7 0.7 0.7];
>> null(NM)
ans =   Null matrix: 4×0
```

Consider β -PIFGs.

```
>> M=[0.2  0  0  0.2 ;0  0.2  0.2  0; 0  0.2  0.3  0; 0.2  0  0  0.3];
>> null(M)
ans = Null matrix: 4×0
>> NM=[0.4   0  0  0.7; 0  0.7  0.7  0; 0  0.7  0.6  0; 0.7  0  0  0.7];
>> null(NM)
ans =  Null matrix: 4×0
```

Theorem 3.5

Every α and β PIFMs are invertible.

Proof: As W.K.T, if a Matrix having determinant other than zero is invertible.

Consider α- PIFM,

```
>> M=[0.2 0.2 0.2 0;0.2 0.2 0 0.2;0.2 0 0.3 0.3;0 0.2 0.3 0.3];
>> DM=det(M)
DM =   -0.0080
>> NM=[0.4 0.7 0.6 0;0.7 0.7 0 0.7;0.6 0 0.6 0.7;0 0.7 0.7 0.7];
>> DNM=det(NM)
```

DNM = -0.5145.Hence the determinant is(-0.0080-0.5145)

Consider β - PIFM,

```
>> M=[0.2 0 0 0.2;0 0.2 0.2 0;0 0.2 0.3 0;0.2 0 0 0.3];
>> DM=det(M)
```

DM = 0.0004

>> NM=[0.4 0 0 0.7;0 0.7 0.7 0;0 0.7 0.6 0;0.7 0 0 0.7];

>> DNM=det(NM)

DNM =0.0147. Determinant for this β -PIFM is (0.0004,0.0147)

Theorem 3.6

Every β and γ PIFMs are singular if $\delta_1{}' \times \delta_1{}''$, $\varepsilon_1{}' \times \varepsilon_1{}''$ are constant and $\delta_2{}' \times \delta_2{}''$, $\varepsilon_2{}' \times \varepsilon_2{}''$ are constant.

Proof: If $\delta_1{}' \times \delta_1{}''$, $\varepsilon_1{}' \times \varepsilon_1{}''$ are constant and $\delta_2{}' \times \delta_2{}''$, $\varepsilon_2{}' \times \varepsilon_2{}''$ are constant, then the IFM corresponding to β and γ- PIFGs are ,

β -PIFM= [(s,t) 0 0 (s,t);0 (s,t) (s,t) 0;0 (s,t) (s,t) 0;(s,t) 0 0 (s,t)]

γ- PIFM =[(s,t) (s,t) (s,t) (s,t); (s,t) (s,t) (s,t) (s,t); (s,t) (s,t) (s,t) (s,t); (s,t) (s,t) (s,t) (s,t)]

From these we clear that ,the two matrices rows are identical.

So D(determinant)=0. Thus β and γ- PIFM are singular.

Theorem 3.7
Any α, β and γ- PIFMs with different Eigen Values are diagonalizable.
Proof: M- any α , β (or)γ PIFM with different eigen-values (a,b) ,where 'a' and 'b' are ,the member value and non-member value of eigen-values . Let the no. of vertices are 2, let (a_1,b_1) , (a_2,b_2) be the eigen-values and (A_1,B_1) , (A_2,B_2) are the two eigen-vectors. W.k.t, a matrix have different eigen-values, then their vectors are not LD . Suppose eigen-vectors are LD. Then $C_1A_1+C_2A_2=0$---------(1)

And $C_1B_1+C_2B_2=0$---------(2) with c_1, c_2 are not both zero.

Consider (1),multiplying both sides by membership eigen-values, we get $C_1a_1A1+C_2a_2A2=0$----------(3)

Multiplying first equation by 'a_1'we get $C_1a_1A_1+C_2a_1A_2=0$----------(4)

Subtracting gives $C_2(a_2-a_1)A_2=0$.W.K.T a_1 and a_2 are different eigen-values. This implies that $C_2=0$. Likewise , $c_1=0$.This gives a contradictory result for linearly dependent.

Likewise, Consider (2). Multiplying both sides by non-membership eigen values, we get $C_1b_1B1+C_2b_2B2=0$----------(5)

Multiplying first equation by 'b_1'we get $C_1b_1B_1+C_2b_1B_2=0$----------(6)

(5)-(6) gives $C_2(b_2-b_1)A_2=0$.Clearly $C_2=0$,likewise $C_1=0$. Which gives the contradiction to statement of LD.

Hence any αlpha, βeta and γamma PIFMS with different eigen-values are diagonalizable.

4.Conclusion

In this journal, matrix representations of α,β and γ - PIFG operations from their IFGs are defined .Algebraic sets and operations are basic for everything .Fuzzy graphs have a numerous applications whose origin is fuzzy relations. We transform every graph as an adjacent matrix. Likewise we change every FGs into FMs which has more real life applications like networking ,cluster analysis,traffic signals etc. Here we gave the relation between IFGMs to linear algebraic concepts like singular, invertible and diagonalizable.

References (APA)

[1]. Fathima Kani B.,Nagoor Gani A.,(2019).Alpha Beta and Gamma Product of Fuzzy Matrices .Bulletin of Pure and Applied Sciences,Vol.38E(Math&Stat.)No.1,359-368.DOI:10.5958/2320-3226.2019.00039.0

[2]. Karunambigai M.G.,Kalaivani O.K.,(2016). Matrix Representations of Intuitionistic Fuzzy Graphs. International Journal of Scientific And Research Publications,Vol.6,Issue 6,520-537.http://www.ijsrp.org/research-paper-0616/ijsrp-p5476.pdf

[3]. Nagoor Gani A., Fathima Kani B. ,(2014) .Alpha Product on Fuzzy Graphs. Advances in Fuzzy Sets and Systems, volume 17, No. 1,pp 27-48. https://www.researchgate.net/publcation/264540454_Alpha_Product_on_Fuzzy_Graphs.

[4]. Nagoor Gani A., Fathima Kani B., (2014) .Beta and Gamma Product of Fuzzy Graphs. International Journal of Fuzzy Mathematical Archive, Vol. 4, No. 120-36. https://www.researchgate.net/publication/277130754_Beta_and_Gamma_Product_of_Fuzzy _Graphs_Intern_J_Fuzzy_Mathematical_Archive_Vol.4_No_1_2014_20-36.

[5]. Sankar Sahoo, Madhumangal Pal, (2015). Different Types of Products of IFGs .Pacific Scientific Review. A: Naional. Science & Engineering,17,87-96. DOI:10.1016/j.psra.2015.12.007.