

Survey of Model Order Reduction in Control System

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ABSTRACT

In this paper, a survey of model order reduction is carried out. It is an important analysis for system identification. Different types of techniques have been discussed in brief. The application of reduction in various fields have been highlighted.

1. Introduction

Model order reduction (MOR) is a technique for reducing the computational complexity of mathematical models in numerical simulations. As such it is closely related to the concept of metamodeling with applications in all areas of mathematical modeling [1].

Many modern mathematical models of real-life processes pose challenges when used in numerical simulations, due to complexity and large size (dimension). MOR aims to lower the computational complexity of such problems, for example, in simulations of large-scale dynamical systems and control systems. By a reduction of the model's associated state-space dimension or degrees-of-freedom, an approximation to the original model is computed which is commonly referred to as a reduced order model [2].

Reduced order models are useful in settings where it is often unfeasible to perform numerical simulations using the complete full order model. This can be due to limitations in computational resources or the requirements of the simulations setting, for instance real-time simulation settings or many-query settings in which a large number of simulations needs to be performed. Examples of real-time simulation settings include control systems in electronics and visualization of model results while examples for a many-query setting can include optimization problems and design exploration. In order to be applicable to real-world problems, often the requirements of a reduced order model.

2. Classification of Model Order Reduction

Figure 1 represents the pictorial of the types of model order reduction. When we perform analysis in time domain it is time domain MOR. When the reduction process is done in frequency domain it is called frequency domain reduction and when both time and frequency domain are utilized, it is generally called as mixed method.

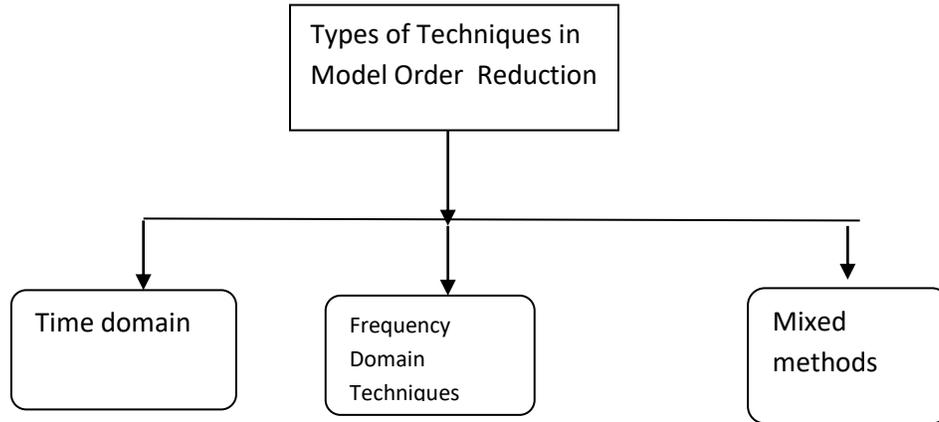


Figure 1. Types of MOR.

3. Some description of MOR techniques

3.1 Davison Technique [3]

A structured approach to the model reduction problem was given by E. J. Davison. The method suggests that the principle of the method is to neglect eigen values of the original system that are farthest from the origin and retain only dominant eigen values and hence dominant time constants of the original system in the reduced order model.

Suppose the original system is represented as

$$\dot{X} = AX + Bu; \quad (1)$$

where A is $n \times n$ matrix, B is

and the new mathematical model is given by

$$Y = A^*Y + B^*u; \quad (2)$$

where $A^* = l \times l$ matrix

3.2 Chidambara Technique

Noting that the basic model of Davison in [4] does not give accurate steady-state response, Chidambara, in his correspondence with Davison had suggested an approach for model order reduction. In this model only the transient response of the left out states are ignored (i.e. X_2), the steady-state contribution of these states are taken into account in order to nullify the steady-state error seen in the basic Davison technique.

3.3 Marshall Technique

S. A. Marshall had proposed an alternate way to compute the reduced order model in [5]. This technique is quite similar to the Chidambara technique since it too takes into account the steady-state values of the X_2 states. The difference exists in the manner in which the reduced order state equation is obtained.

4. Application of Model Reduction in control system

4.1 Space-time kinetics model of AHWR

Application of model order reduction techniques based on Davison's and Marshall's dominant mode retention and model decomposition into slow and fast subsystems based on Singular Perturbation analysis have been successfully explored for AHWR. Davison's and Marshall's dominant mode retention techniques require diagonalization of the original 80th order model while Singular Perturbation techniques requires reordering of state variables and block-diagonalization [5].

4.2 Electrical engineering

Model Order Reduction Techniques focuses on model reduction problems with particular applications in electrical engineering. Starting with a clear outline of the technique and their wide methodological background, central topics are introduced including mathematical tools, physical processes, numerical computing experience, software developments and knowledge of system theory. Several model reduction algorithms are then discussed. The aim of this work is to give the reader an overview of reduced-order model design and an operative guide.

4.3. Finite Space time

Despite the continued rapid advance in computing speed and memory the increase in the complexity of models used by engineers persists in outpacing them. Even where there is access to the latest hardware, simulations are often extremely computationally intensive and time-consuming when full-blown models are under consideration.

The need to reduce the computational cost involved when dealing with high-order/many-degree-of-freedom models can be offset by adroit computation. In this light, model-reduction methods have become a major goal of simulation and modeling research. Model reduction can also ameliorate problems in the correlation of widely used finite-element analyses and test analysis models produced by excessive system complexity [6].

4.4 Medical line

Silico analysis tools in the bio-medical field are moving from the research context to the patient specific treatment and prevention one. Hemo-dynamics is receiving a great attention and an accurate CFD modelling can be adopted to produce a digital medical twin capable to reliably predict pathology the evolution and the effect of surgical corrections. The availability of in silico digital twins based on CAE simulations is one of the key enablers; parametric shape of vessels and reduced order models (ROM) are a promising solution. The ROM approach requires HPC to be built, but it can be consumed almost in real time and outside from the standard CAE tools as well. In this paper the concept is demonstrated exploiting the new ROM Builder available in ANSYS 19.1. We developed a pipeline for the aortic aneurysm to study the effect of the bulge shape progression on the flow field [7].

4.5. Application in health monitoring of plate using Lamb wave propagation and impedance method

In this paper, a new method based on global kernel k -means clustering (GKKMC) has been developed as model order reduction to model electromechanical impedance and Lamb wave in a plate structure. First, a model, based on the spectral finite element method, is developed to simulate piezoelectric wafer active sensor (PWAS) induced electromechanical impedance and Lamb wave propagation. Second, the related coupled electromechanical field equations are solved in 3 dimensions, then an electric voltage signal is

applied to PWAS actuator, and finally the produced voltage in PWAS sensor is calculated. In reality, high frequency impedance and Lamb wave simulation in a plate need high degrees of freedom, which leads to a very slow simulation in time and frequency domain calculations. To overcome this problem, GKKMC as a model order reduction approach is proposed and applied.

4.6. Parametric model order reduction and its application to inverse analysis of large nonlinear coupled cardiac problem

Predictive high-fidelity finite element simulations of human cardiac mechanics commonly require a large number of structural degrees of freedom. Additionally, these models are often coupled with lumped-parameter models of hemo dynamics. High computational demands, however, slow down model calibration and therefore limit the use of cardiac simulations in clinical practice. As cardiac models rely on several patient-specific parameters, just one solution corresponding to one specific parameter set does not at all meet clinical demands. Moreover, while solving the nonlinear problem, 90% of the computation time is spent solving linear systems of equations. We propose a novel approach to reduce only the structural dimension of the monolithically coupled structure-wind kessel system by projection onto a lower-dimensional subspace. We obtain a good approximation of the displacement field as well as of key scalar cardiac outputs even with very few reduced degrees of freedom while achieving considerable speedups.

4.7. Applications in Finite Element Analysis

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The need to reduce the computational cost involved when dealing with high-order/many-degree-of-freedom models can be offset by adroit computation. In this light, model-reduction methods have become a major goal of simulation and modeling research. Model reduction can also ameliorate problems in the correlation of widely used finite-element analyses and test analysis models produced by excessive system complexity [8].

4.8 FOR INDUCTION MACHINES

This paper investigates the accuracy of the liberalized reduced order models of induction machines as applied to dynamic stability studies. These machines are represented in detail by a fifth or seventh order model. The order of the model is reduced by neglecting the stator transients. However, the reduced model does not yield the same eigen values for the retained modes. The deviation from the full model eigen values becomes more noticeable for small horsepower motors. It is shown that the accuracy can be restored through the use of correction factors to compensate for the dynamics of the neglected transients. These factors are concluded from the selective modal analysis which provides full information on the participation of the less relevant states in the retained modes and thus allows for the compensation for their dynamics, whenever neglected, in the retained states [9].

4.9. Model order reduction in aerodynamics

The need of the aerospace industry, at national or European level, of faster yet reliable computational fluid dynamics models is the main drive for the application of model reduction techniques. This need is linked to the time cost of high-fidelity models, rendering them inefficient for applications like multi-disciplinary optimization. With the goal of testing and applying model reduction to computational fluid dynamics models applicable to lifting surfaces, a bibliographical research covering reduction of nonlinear, dynamic, or steady models was conducted. This established the prevalence of projection and least mean

squares methods, which rely on solutions of the original high-fidelity model and their proper orthogonal decomposition to work [10].

Conclusion

In this paper we made a survey of model order reduction. Few methods have been described and application is presented. In future, MOR techniques for control-oriented applications will be performed.

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